A Common Value Auction with Bidder Solicitation

Search and Switching Cost Workshop, Moscow

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- Two objectives:
  - Some understanding of equilibrium in this environment
  - Revisit information aggregation in large auction (Milgrom 1979)
    When solicitation costs are small, auction is endogeneously large

Alternative Interpretation

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Simultaneous ("Noisy") Search
Our model can be interpreted as a batch search model as in Burdett-Judd (1983), with the added feature of adverse selection.

Model (1): Seller and Buyers

- A single seller and $\bar{N}$ buyers
- Seller's cost $c = 0$ is commonly known
- Seller's type $w \in \{L, H\}$; prior probabilities ρ_L and ρ_H
- Buyers have common values,

$$v_w \in \{v_L, v_H\}, \qquad c \leq v_L < v_H$$

- w is private information of the seller

Model (2): Signal Distribution

- Each buyer observes signal $x \in [\underline{x}, \bar{x}]$
  - conditional on type w, signals are independent and identically distributed
  - atomless c.d.f. G_w admits a density g_w that is strictly positive on $[\underline{x}, \bar{x}]$
- Likelihood ratio $\frac{g_H(x)}{g_L(x)}$ is weakly increasing;
  likelihood ratio is right-continuous at $\underline{x}$ and left-continuous on $(\underline{x}, \bar{x}]$
- Most favorable signal is $\bar{x}$. Signals boundedly informative,

$$0 < \frac{g_H(\underline{x})}{g_L(\underline{x})} < 1 < \frac{g_H(\bar{x})}{g_L(\bar{x})} < \infty$$

Model (3): First Price Auction with Bidder Solicitation

1. Seller knows w; solicits n randomly drawn bidders at marginal cost $s > 0$, with $n \in \{1, ..., \bar{N}\}$, $\bar{N} \geq \frac{v_H}{s}$.
2. Each solicited bidder privately observes a signal $x \sim G_w$
   w and n unobservable to buyers
3. Solicited bidders submit bids simultaneously.
4. Highest bidder wins; ties are broken randomly

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Payoffs when p is the winning bid:
Winning Bidder: $v_w - p$; Other Buyers: 0; Seller: $p - c - ns$

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- Study equilibrium winning bid when s is small.

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$$\frac{\rho_H g_H(x) \frac{n_H}{\bar{N}}}{\rho_L g_L(x) \frac{n_L}{\bar{N}} + \rho_H g_H(x) \frac{n_H}{\bar{N}}} = \frac{\frac{\rho_H}{\rho_L} \frac{g_H(x)}{g_L(x)} \frac{n_H}{n_L}}{1 + \frac{\rho_H}{\rho_L} \frac{g_H(x)}{g_L(x)} \frac{n_H}{n_L}}$$

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- The ratio $\frac{n_H}{n_L}$ captures "solicitation effect"

- Solicitation is bad news ("curse") if $\frac{n_H}{n_L} < 1$

Bidding Equilibrium

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A symmetric and pure **bidding equilibrium** given solicitation strategy (n_L, n_H) is a bidding strategy $\beta : [\underline{x}, \bar{x}] \rightarrow \mathbb{R}$ such that for all $x \in [\underline{x}, \bar{x}]$, $b = \beta(x)$ maximizes interim expected payoffs.

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Equivalent to equilibrium of standard common value auction if $n_H = n_L = n$

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- Values $v_L = 0$ and $v_H = 1$; Uniform prior, $\rho_H = \rho_L = \frac{1}{2}$
- Signals $x \in [\underline{x}, \bar{x}] = [0, 1]$
- $g_H(x) = 0.8 + 0.4x$ and $g_L(x) = 1.2 - 0.4x$

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Claim: Let $\bar{N} = 10$ and solicitation strategy $n_L = 6$ and $n_H = 2$. There is a bidding equilibrium in which

$$\beta(x) = 0.4 \qquad \forall x \in [\underline{x}, \bar{x}].$$

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- Interim expected value conditional on $\bar{x} = 1$ and being solicited:

$$\frac{1}{1 + \frac{\rho_H}{\rho_L}\frac{g_H(\bar{x})}{g_L(\bar{x})}\frac{n_H}{n_L}} v_L + \frac{\frac{\rho_H}{\rho_L}\frac{g_H(\bar{x})}{g_L(\bar{x})}\frac{n_H}{n_L}}{1 + \frac{\rho_H}{\rho_L}\frac{g_H(\bar{x})}{g_L(\bar{x})}\frac{n_H}{n_L}} v_H = 0 + \frac{\frac{3}{2}\frac{2}{6}}{1 + \frac{3}{2}\frac{2}{6}}(1) = \frac{1}{3}$$

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- The solicitation effect offsets the informational content of the signal:

$$\frac{n_H}{n_L}\frac{g_H(\bar{x})}{g_L(\bar{x})} = \frac{2}{6}\frac{3}{2} = \frac{1}{2}$$

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- Generally, solicitation curse is "overwhelming" if

$$\frac{g_H(\bar{x})}{g_L(\bar{x})}\frac{n_H}{n_L} < 1.$$

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- Expected value conditional on $\underline{x}$, conditional on being solicited, and conditional on winning at $p^* = 0.4$ is

$$\frac{1}{1+\frac{\rho_H}{\rho_L}\frac{g_H(\underline{x})}{g_L(\underline{x})}\frac{n_H}{n_L}\frac{\pi_H(p^*)}{\pi_L(p^*)}}v_L + \frac{\frac{\rho_H}{\rho_L}\frac{g_H(\underline{x})}{g_L(\underline{x})}\frac{n_H}{n_L}\frac{\pi_H(p^*)}{\pi_L(p^*)}}{1+\frac{\rho_H}{\rho_L}\frac{g_H(\underline{x})}{g_L(\underline{x})}\frac{n_H}{n_L}\frac{\pi_H(p^*)}{\pi_L(p^*)}}v_H = \frac{\frac{2}{3}\frac{2}{6}\frac{\frac{1}{2}}{\frac{1}{6}}}{1+\frac{2}{3}\frac{2}{6}\frac{\frac{1}{2}}{\frac{1}{6}}}1 = 0.4$$

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- Thus, $\underline{x}$ expects (weakly) positive payoffs from bidding 0.4.

- Winning is "good news," $\frac{\Pr[Win|H]}{\Pr[Win|L]} = \frac{\frac{1}{n_H}}{\frac{1}{n_L}} = \frac{\frac{1}{2}}{\frac{1}{6}} = 3$.
- Bidding in Atoms provides insurance ("hiding in the crowd") given uniform tie-breaking rule.

Observations

- Whenever $\frac{n_H}{n_L} = \frac{1}{3}$ and $n_H \geq 2$ there is a bidding equilibrium where all bidders bid $\bar{b} \in [1/3, 0.4]$.

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- Construction is not an equilibrium. Seller's solitication strategy not optimal.

Equilibrium

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A symmetric and **pure strategy equilibrium** consists of a bidding strategy $\beta : [\underline{x}, \bar{x}] \to \mathbb{R}$ and a solicitation strategy (n_L, n_H) such that

(i) β is a bidding equilibrium given solicitation strategy (n_L, n_H);
(ii) solicitation is optimal,

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An **equilibrium** (without qualifier) allows for mixed solicitation strategy, denoted $\eta_w \in \Delta\left\{1, ..., \bar{N}\right\}$.

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$$\frac{g_H(x)}{g_L(x)} = \begin{cases} \frac{g_H(\bar{x})}{g_L(\bar{x})} & \text{if} \quad x \geq \hat{x} \\ \frac{g_H(\underline{x})}{g_L(\underline{x})} & \text{if} \quad x \leq \hat{x} \end{cases}$$

Pooling with Bidder Solicitation

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- Good News/Bad News: $g_H(x)/g_L(x)$ constant on $[0, \hat{x}]$, $(\hat{x}, \bar{x}]$
- Symmetric Signals: $G_L(\hat{x}) = 1 - G_H(\hat{x})$
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Proposition. Complete Pooling Possible in the Limit
Suppose signals are as described before. Then, for all $\{s^k\}$, $\lim_{k\to\infty} s^k = 0$, there exists a sequence of equilibria $\{\beta^k, \eta^k\}$ such that

$$\lim_{k\to\infty} E\left[p|\eta_H^k, H, \beta^k\right] = \lim_{k\to\infty} E\left[p|\eta_L^k, L, \beta^k\right] < \rho_L v_L + \rho_H v_H.$$

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- Auction does not become "competitive"
- $G_H(\hat{x})$ can be arbitrarily small, i.e., signals can be arbitrarily informative

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- Bidding is step function,

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- With bids $\underline{b}^k < \bar{b}$, and

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- When $s^k \to 0$, solicitation strategy such that both types
  - solicit unboundedly many bidders
  - end up trading almost surely at $\bar{b}$

Pooling: Sampling

- **Idea:** Given step-function, soliciation strategy (n_H^k, n_L^k) is optimal if

$$\left(G_L\left(\hat{x}\right)\right)^{n_L^k}\left(1-G_L\left(\hat{x}\right)\right)\left(\bar{b}-\underline{b}^k\right) \quad = \quad s^k$$

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- This implies

$$n_L^k \ln G_L\left(\hat{x}\right) + \ln\left(1-G_L\left(\hat{x}\right)\right) = n_H^k \ln G_H\left(\hat{x}\right) + \ln\left(1-G_H\left(\hat{x}\right)\right)$$

- Taking limits and re-ordering

$$\lim_{k\to\infty} \frac{n_H^k}{n_L^k} = \frac{\ln G_L\left(\hat{x}\right)}{\ln G_H\left(\hat{x}\right)}$$

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- Taking limits and re-ordering

$$\lim_{k\to\infty}\frac{n_H^k}{n_L^k}=\frac{\ln G_L\left(\hat{x}\right)}{\ln G_H\left(\hat{x}\right)}<1$$

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- Taking limits and re-ordering

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- From $\frac{g_H(\bar{x})}{g_L(\bar{x})} = \frac{1 - G_H(\hat{x})}{1 - G_L(\hat{x})}$,

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n_L^k \ln G_L\left(\hat{x}\right)+\ln\left(1-G_L\left(\hat{x}\right)\right)=n_H^k \ln G_H\left(\hat{x}\right)+\ln\left(1-G_H\left(\hat{x}\right)\right)
$$

- Taking limits and re-ordering

$$
\lim_{k\to\infty}\frac{n_H^k}{n_L^k}=\frac{\ln G_L\left(\hat{x}\right)}{\ln G_H\left(\hat{x}\right)}<1
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$$

Pooling: Sampling

- **Idea:** Given step-function, soliciation strategy $\left(n_H^k, n_L^k\right)$ is optimal if
$$\begin{aligned}\left(G_L\left(\hat{x}\right)\right)^{n_L^k}\left(1-G_L\left(\hat{x}\right)\right)\left(\bar{b}-\underline{b}^k\right) &= s^k \\ \left(G_H\left(\hat{x}\right)\right)^{n_H^k}\left(1-G_H\left(\hat{x}\right)\right)\left(\bar{b}-\underline{b}^k\right) &= s^k\end{aligned}$$

- This implies
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- This last observation holds generally.

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- Observe that

$$\lim_{k\to\infty} E[v|\text{solicited, signal } \bar{x}, \text{ win at } \bar{b}] = \rho_L v_L + \rho_H v_H$$

Separation with Bidder Solicitation

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Proposition. Full Separation Possible in the Limit.

Suppose signals are either good news or bad news. For every $\varepsilon > 0$, there is some $R^{SOL}(\varepsilon)$ such that whenever $\frac{g_H(\bar{x})}{g_L(\bar{x})} \geq R^{SOL}(\varepsilon)$ and $\{s^k\} \to 0$, there exists a̲ sequence of equilibria (β^k, η^k) such that

$$\begin{aligned} \lim_{k\to\infty} \inf E\left[p|\eta_H^k, H, \beta^k\right] &\geq v_H - \varepsilon, \\ \lim_{k\to\infty} \sup E\left[p|\eta_L^k, L, \beta^k\right] &\leq v_L + \varepsilon. \end{aligned}$$

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Conclusion

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- Endogenous relationship between value and number of bidders: Identified "*solicitation curse*"
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- Multiple limit outcomes (in a "two-signal" example):
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- What happens when number of solicited bidders observable? Incentive to signal? Trade-offs?

Bidding Strategies are Weakly Monotone

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Lemma. Given any solicitation strategy (η_L, η_H) such that each type solicits at least two bidders, i.e., $\eta_L(1) = \eta_H(1) = 0$. Then, in every bidding equilibrium, $\frac{g_H(x_1)}{g_L(x_1)} > \frac{g_H(x_2)}{g_L(x_2)}$, implies $\beta(x_1) \geq \beta(x_2)$ for almost all x_1, x_2.

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Counterexample: Suppose $\eta_H(1) = 1$ and $\eta_L(1) = 0$. Then, in every bidding equilibrium, $\frac{g_H(x_1)}{g_L(x_1)} > \frac{g_H(x_2)}{g_L(x_2)}$, implies $\beta(x_1) < \beta(x_2)$ for almost all x_1, x_2.

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Intuition: Consider $\frac{g_H(\underline{x})}{g_L(\underline{x})} = 0$ and $\frac{g_H(\bar{x})}{g_L(\bar{x})} = \infty$