Ordered Search with Asymmetric Product Design

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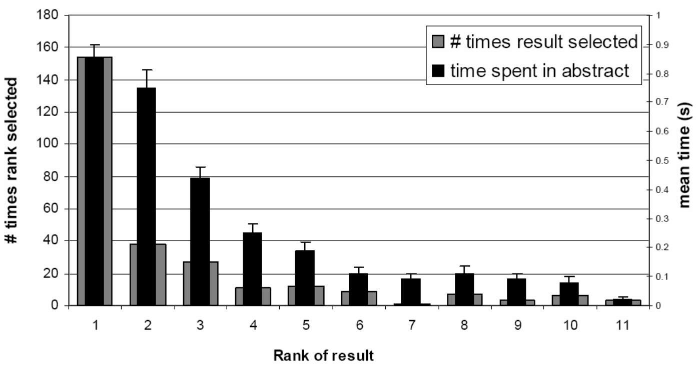
Stylized facts about consumers' reading habits show that:

- Most consumers do read books (newspaper, etc.) in the top-down order.
- Items do receive different levels of attention depending on their rankings.
- Granka, Joachims and Gay 2004 employs eye-tracking analysis to explore both the click and attention distribution from Google.



Introduction

- Products designed for a large population are called generic goods, which provide this population with a stable level of surplus.
- Products designed for a small population are called niche goods, which provide a market niche with a higher level of surplus, and the rest of the market with a lower level of surplus.
- Example:

ripped jeans:niche

standard jeans:generic

Our main research question is,

- Which product order maximizes industry profit?
- Examples: on-line store, supermarket, salesman etc.

Short Answers

- The niche-generic ordering increases industry profit.
- The same order is preserved by strategic consumers when search cost and consumer heterogeneity ratios are at some parameter ranges.

Roadmap

Related Literatures

- Product Search: Wolinsky (1986) and Anderson-Renault (1999)
- Prominence: Armstrong-Vickers-Zhou (2009)
- Ordered search: Arbatskaya (2007), Zhou (2011)
- Design Choice: Johnson-Myatt (2006), Larson (2011), Isaac-Caruana-Cuñat (2010)

Framework

1. There is one unit mass of consumers, each has unit demand. Consumer i's utility from product j, $j \in \{N, G\}$ follows a linear random utility model (Wolinsky 1986, Anderson and Renault 1999):

$$u_{ij} = \underbrace{\mu_j \epsilon_{ij}}_{product\,match} - \underbrace{p_j}_{price\,paid}$$

2. ϵ_{ij} is a measure of idiosyncratic match, preference or the consumer's location independently and identically distributed under some distribution function F.

Assumption 1: F is the uniform distribution with support $[-1, 1]$. (symmetry)

Consumer Heterogeneity (preference intensity): a firm-specific measure of the importance of the role the match plays on buying decisions. In Larson (2011), Firm j produces niche (generic) goods if μ_j is large (small).

- Firm N (Firm G) offers niche goods (generic goods), therefore $\mu_N > \mu_G$. Each firm knows whether it is selling niche or generic goods.

Framework

GAME Structure: one-shot game with three stages

1. **Firm Profit:** firms observe prices but no match information. Denote firm j's profit function by
$$\Pi_j^{firm\,k \prec firm\,l}(p_N, p_G)$$
2. **Consumer Surplus:**
$$u_{ij}(p_j) - kc$$
3. **Timing:**
Stage 1) The planner (platform, multi-product firm) chooses product order. Stage 2) Firms observe their locations and simultaneous choose prices. Stage 3) Consumers receive the information from the top firm for free and make their search and purchase decisions simultaneously.
$$\text{consumers} \Rightarrow \underset{\Downarrow_{\text{trade}}}{\text{top firm}} \Leftrightarrow \text{second firm} \Rightarrow \text{trade}$$
4. **Equilibrium:** Perfect Bayesian Nash Equilibrium (PBNE).

Imperfect information

Assumption 2: Market is covered by two firms: if a consumer does not buy at all, her surplus is $-\infty$.

Search cost: in practice, price and product information is available only by searching them costly. These costs are usually transportation cost, time cost and opportunity cost. The search cost is identical for each consumer and for each time of search.

1. **Assumption 3**: consumers can not choose their searching paths which are identical for everyone and predetermined by the planer. (restricted order)
2. **Assumption 4**: after sampling one firm, the consumer can recall that firm at any time at no cost. (free recall)

When to stop?

- Consumers based their search decisions on the current offer, price expectations and the search cost c. Suppose firm j $\prec$ firm j', $\mathrm{u_{ij}}$ is the current offer and $\mathrm{p_{j'}}$ is the price expectation.
- Let $\hat{\epsilon}_j$ solve the following equation:

$$\underbrace{\int_{u_{ij'} \geq u_{ij}} \left(\mu_{j'} \epsilon_{j'} - \mu_j \hat{\epsilon}_j + p_j - p_{j'} \right) \frac{\mathrm{d}\epsilon_{j'}}{2}}_{expected\, incremental\, utility} = \underbrace{c}_{search\, cost} \tag{1a}$$

$$\Rightarrow \hat{\epsilon}_j(p_j, p_{j'}) = (\mu_{j'} + p_j - p_{j'} - 2\sqrt{\mu_{j'} c})/\mu_j \tag{1b}$$

$$search\, if\, \epsilon_j < \hat{\epsilon}_j ; purchase\, if\, \epsilon_j > \hat{\epsilon}_j$$

Lemma 1: *the threshold for searching exists uniquely, increases in price of the current offer and consumer heterogeneity of the second firm, and decreases in consumer heterogeneity of the current offer and expected price of the second firm.*

Equilibria

Assumption 5 (imperfect information): the search cost is larger than the maximum search cost leading to full information, and smaller than the minimum search cost preventing every consumer from searching the second firm. $\mathrm{c}_{\max} > \mathrm{c} > \mathrm{c}_{\min}$, where

$$\mathrm{c}_{\max} = \min_{j} \left((\mu_{j'} + \mu_j + p_j - p_{j'})/2 \right)^2 / \mu_{j'},\ j \in \{N, G\}\, and\, j' \neq j$$

$$\mathrm{c}_{\min} = \max_{j} \left((\mu_{j'} - \mu_j + p_j - p_{j'})/2 \right)^2 / \mu_{j'},\ j \in \{N, G\}\, and\, j' \neq j$$

Equilibria

Proposition 1: *in the generic-niche ordering, the unique existing pair of price equilibrium satisfies* $\Delta > 0$, $\Delta + \Delta_\mu > 0$ *and the following.*
$p_N = (\mu_N + \mu_G - \Delta + 2\sqrt{\mu_N c})/2$, $p_G = (4\mu_N(\mu_G + c + \Delta) - (\Delta_\mu + \Delta)^2)/2(\mu_N + \mu_G - \Delta)$

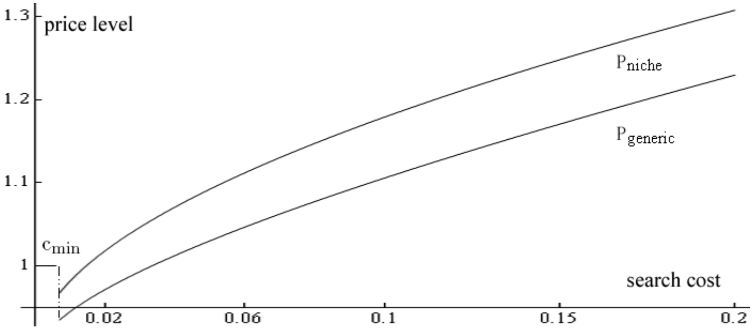
- Price difference $\Delta = p_N - p_G$ is determined by,

$$\Delta = \frac{(\mu_N + \mu_G - \Delta + 2\sqrt{\mu_N c})}{2} - \frac{(4\mu_N(\mu_G + c + \Delta) - (\Delta_\mu + \Delta)^2)}{2(\mu_N + \mu_G - \Delta)}$$

- The top firm sets a lower price than the second firm. Prices $p_j(\mu_N, \mu_G, c)$ and price differences satisfy $p_j(\alpha\mu_N, \alpha\mu_G, \alpha c) = \alpha p_j$ and $\Delta(\alpha\mu_N, \alpha\mu_G, \alpha c) = \alpha\Delta$.

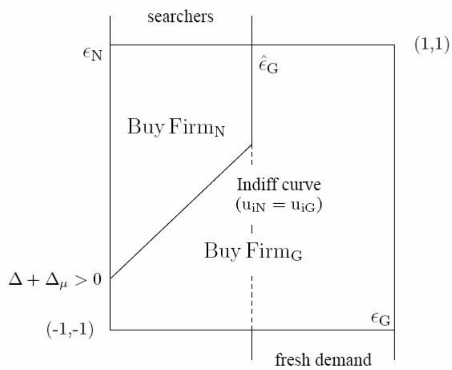
Equilibria

Figure: Price Continuity when generic firm $\prec$ niche firm.



Equilibria

Figure: Demand share at the generic-niche ranking.



Equilibria

Proposition 2: *in the niche-generic ordering, two existing price equilibria satisfy* $\Delta < 0$ *and respectively the followings.*

- *Equilibrium 1 satisfies* $p_N = \mu_N + c - \Delta$, $p_G = \mu_G(\mu_N + \Delta - c)/(\mu_G - \sqrt{\mu_G c})$ *and* $\Delta + \Delta_\mu > 0$.

$$\Delta = \left(\mu_N + c - \Delta\right) - \frac{\mu_G(\mu_N + \Delta - c)}{(\mu_G - \sqrt{\mu_G c})}$$

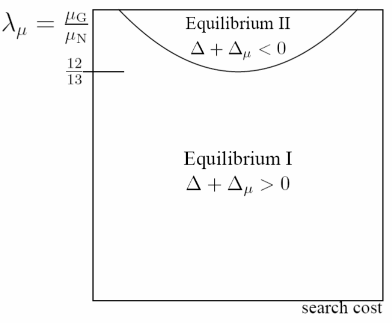
- *Equilibrium 2 satisfies* $p_N = (4\mu_G(\mu_N + c - \Delta) - (\Delta + \Delta_\mu)^2)/2(\mu_G + \mu_N + \Delta)$, $p_G = (\mu_N + \mu_G + \Delta + 2\sqrt{\mu_G c})/2$ *and* $\Delta + \Delta_\mu < 0$.

$$\Delta = \frac{(4\mu_G(\mu_N + c - \Delta) - (\Delta + \Delta_\mu)^2)}{2(\mu_G + \mu_N + \Delta)} - \frac{(\mu_N + \mu_G + \Delta + 2\sqrt{\mu_G c})}{2}$$

- *The top firm sets a lower price than the second firm.* $p_j(\alpha\mu_N, \alpha\mu_G, \alpha c) = \alpha p_j$ *and* $\Delta(\alpha\mu_N, \alpha\mu_G, \alpha c) = \alpha\Delta$.

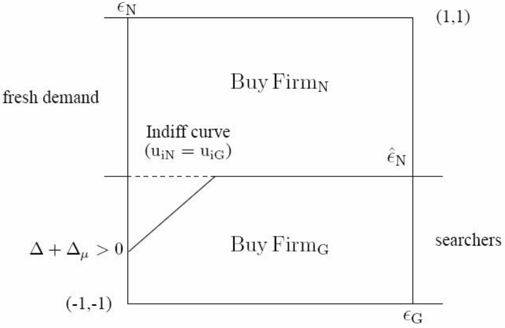
Equilibria

Figure: Equilibria under the niche-generic ranking.



Equilibria

Figure: Demand share at the niche-generic ranking.



Comparative statics

Proposition 3: *the niche firm receives a higher profit in the niche-generic order than the reverse order.* (similar to Zhou 2011)

Lemma 2: *under assumption 1-5, the generic-niche ordering induces more searching than the reverse order if search cost is not too large ($\mu_G \gg c$).*

Table: Effect of different orders on percentage of searchers at c=1/8.

μ_G/μ_N	niche-generic	generic-niche	μ_G/μ_N	niche-generic	generic-niche
0.3	31.7%	68.8%	0.6	46.3%	66.4%
0.4	37.6%	68.9%	0.7	50.2%	65%
0.5	42.2%	67.8%	0.8	54%	63.5%

Comparative statics

Proposition 4: *there exists a cut-off value $\tilde{\mu}$ such that the industry profit is higher in niche-generic ranking if $\lambda_\mu = \mu_{\mathrm{G}}/\mu_{\mathrm{N}} < \tilde{\mu}$.*

$$\Pi^{niche \prec generic} = \frac{\mu_G(\mu_N + \Delta^{N-G} - c)^2}{2(\mu_G - \sqrt{\mu_G c})} + \frac{(\mu_N - \Delta^{N-G} - c)^2}{2}$$

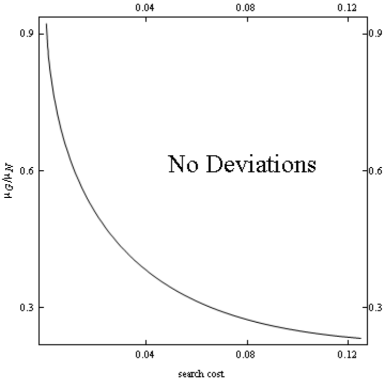
$$\begin{aligned}\Pi^{generic \prec niche} &= \frac{(4\mu_N(\mu_G + \Delta^{G-N} + c)) - (\mu_N - \mu_G + \Delta^{G-N})^2)^2}{16\mu_G(\mu_N + \mu_G - \Delta^{G-N})} \\ &= \frac{(\mu_N + \mu_G - \Delta^{G-N} + 2\sqrt{\mu_N c})^2}{16\mu_G}(\mu_N + \mu_G - \Delta^{G-N} - 2\sqrt{\mu_N c})\end{aligned}$$

Table: Effect of different orders on industry profit at c=1/8.

μ_G/μ_N	niche-generic	generic-niche	μ_G/μ_N	niche-generic	generic-niche
0.3	0.892	0.474	0.6	1.56	1.25
0.4	1.12	0.71	0.7	1.77	1.55
0.5	1.33	0.97	0.8	1.99	1.87

Proposition 5: *there exist two cut-off values* $\tilde{c}$ *and* $\underline{\mu}$ *such that, when* $\tilde{c} < c \leq \lambda_\mu, \underline{\mu} < \lambda_\mu < \tilde{\mu}$, *there exists a PBNE where (1) both firms hold the belief that consumers search firms in the niche-generic order. (2) Consumers search firms in the niche-generic order.*

Figure: Deviation incentives by consumers.



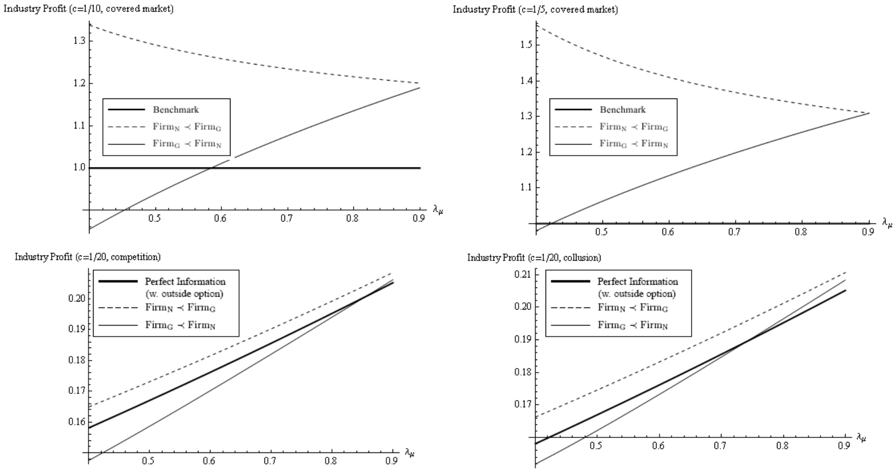
Simulation

- **Assumption 1-5**
- **Assumption 6**: any consumer purchases a good only if the good offers him positive surplus: $\mathrm{u_{ij}} \geq 0$ for $\mathrm{j} \in \{\mathrm{N}, \mathrm{G}\}$.
- **Assumption 7**: $\mathrm{c_{min}} < \mathrm{c} \leq \min\{\mu_j/16\}$.
  - Consumers receiving negative surplus from the top firm search the second firm so that we avoid the existence of consumers who do not purchase and do not search.
- **Market structures**:
  - Competition: the planer is the third party who make the arrangement decision. Two firms make price choices independently from each other.
  - Collusion: the planer is a multi-product firm.

Simulation

- Duopoly Model
  - Covered Market
    1.Perfect Information
    2.Generic-Niche
    3.Niche-Generic
  - With Outside Option (set to zero)
    - Competition
      1. Perfect Information
      2. Generic-Niche
      3. Niche-Generic
    - Collusion
      1. Perfect Information
      2. Generic-Niche
      3. Niche-Generic

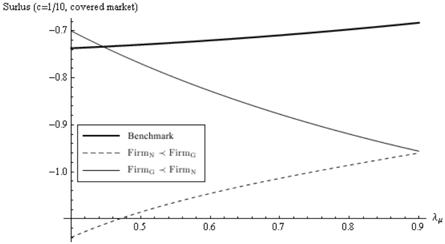
Industry Profit

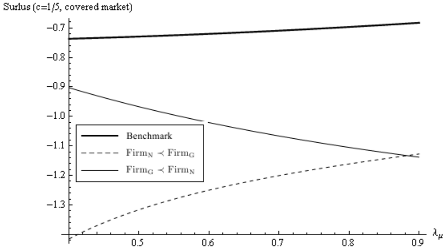


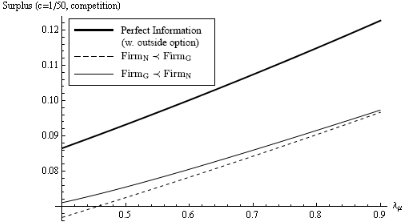
Determinants: price-level

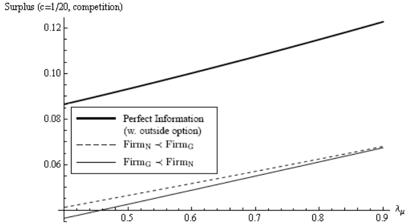
Consumer Surplus

- Recall that a consumer's surplus is its utility minus the search cost he has incurred: $u_{ij} - k\,c$.









Determinants: price-level, match

Social Welfare

- Distorted search rule under perfect (imperfect) information

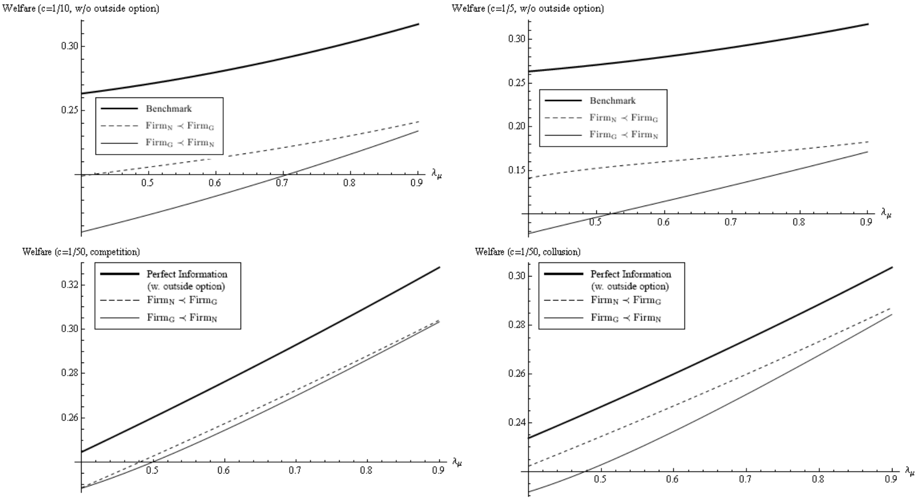
  1. Perfect information: $\hat{\epsilon}_j = \min(\mu_{j'}, \mu_j)/\mu_j$
  2. Imperfect information: $\hat{\epsilon}_j(p_j, p_{j'}) = (\mu_{j'} + \underbrace{p_j - p_{j'}}_{price\ distortion} - \underbrace{2\sqrt{\mu_{j'} c}}_{search\ cost})/\mu_j$

- Niche-generic order distorts more the optimal search rule.

Table: Values of $p_j - p_{j'} - 2\sqrt{\mu_{j'} c}$ from different orders at c=1/8.

μ_G/μ_N	niche-generic	generic-niche	μ_G/μ_N	niche-generic	generic-niche
0.3	-0.666	-0.887	0.6	-0.674	-0.803
0.4	-0.648	-0.849	0.7	-0.696	-0.791
0.5	-0.656	-0.822	0.8	-0.720	-0.783

Social Welfare



Determinants: optimal search rule

Conclusion

1. Prices increase along the searching path, and can decrease in consumer heterogeneity if this induces more intense competition.
2. The niche-generic order increases industry profit by dampening price competition.
3. The same order is preserved by consumers for some parameter ranges, and increases social welfare.