Nonreservation-price Equilibria

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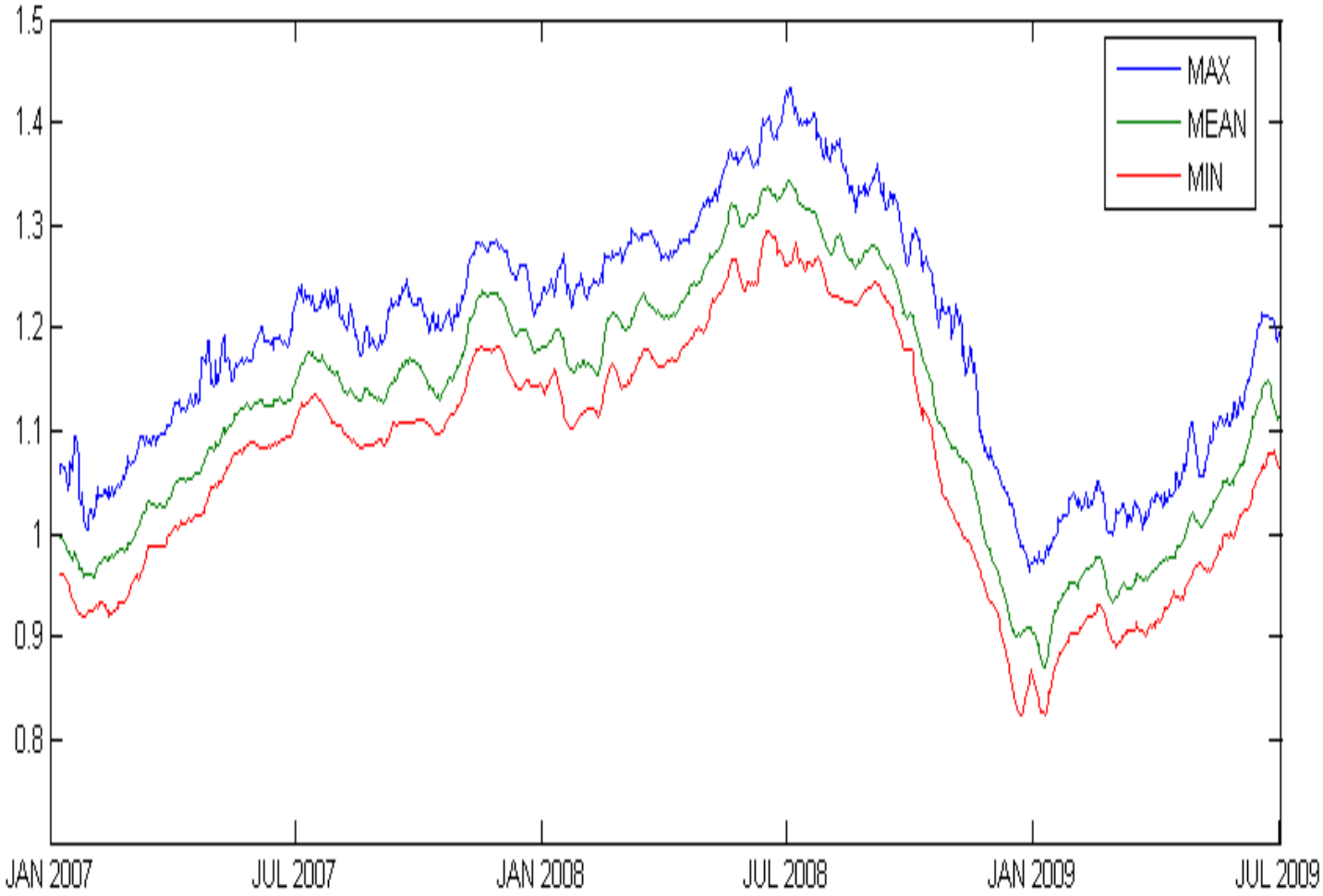
Euro-super 95 prices in Vienna



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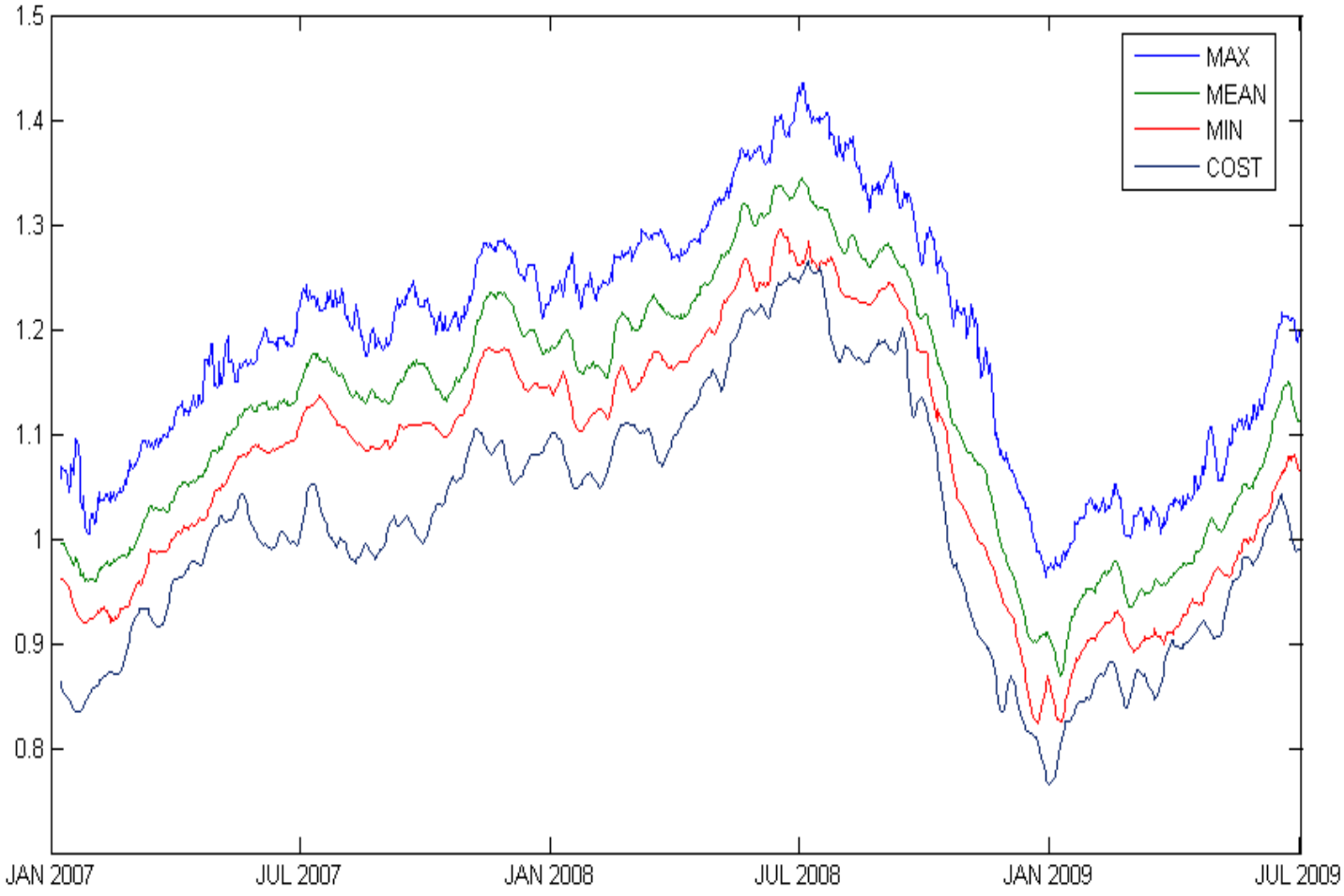

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Reservation prices

- Consumer decides to buy at current price or continue to search. At reservation price indifferent.
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- Consumers know firms' cost and expectation based on firms' strategies
  - This is what most of the consumer search literature does
  - But, if consumers do not know prices, it is far-fetched to assume they know cost
- Consumers do not know cost and believe that firms' prices are independent of cost
  - Not feasible as in (almost) all models, firms' optimal prices depend on their cost

Motivation

- There is a literature on consumer search with uncertain cost (Benabou and Gertner (1993), Dana (1994), Fishman (1996), Tapatta (2009), Janssen, Pichler and Weidenholzer (2011))
- Sequential search along the lines of Stahl (1989, AER)
  - oligopoly, homogenous good, common production cost
- Information asymmetry: Incompletely informed consumers do not know firms' production cost
  - Consumers observe price, update beliefs about underlying cost and then decide whether to buy or to continue to search
- Literature still considers equilibria where consumers' strategy is characterized by a reservation price

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- Literature shows that compared to the case where consumers know firms' cost, consumers suffer from cost uncertainty

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- Shows that in any non-reservation price equilibrium, there is active search
- Intends to show that a non-reservation price equilibrium always exist
- Shows that compared to the case where consumers know firms' cost, consumers may be better off under cost uncertainty (as it induces them to actively (and rationally) search, which puts downward pressure on prices)
- Implies new challenge for empirical research on consumer search

Literature

- Diamond (1971, JET), Rothschild (1974, JPE)
  - Search with unknown but **exogenous** price distribution
- Stahl (1989, AER)
  - Sequential search with an **endogenous** price distribution and **complete information**
- Benabou and Gertner (1993, RESTUD), Dana (1994, IER)
  - **Non-sequential** search under **incomplete information**
- Fishman (1996, QJE), Yang and Ye (2008, RAND), Tappata (2009, RAND)

The model

- 2 firms
  - produce homogenous good at marginal cost c, compete in prices
  - c is either high or low: c_H, c_L with prob. α, $1-\alpha$
  - realization of c identical for all firms
- $[0,1]$ consumers
  - unit demand and valuation v; payoff $v-p$
  - search for low prices to maximize payoff
  - observe 1st price for free, pay search cost s to obtain 1 additional price quote (sequential search)
  - fraction λ are shoppers ($s=0$); $1-\lambda$ non-shoppers ($s>0$)
  - **consumers do not observe realized** c

Reservation Price Equilibria

Firms

- Let $F(p|c)$ be the cumulative distr. corresponding to $f(p|c)$
- Expected profit when setting p:

$$\pi(p, c, F) = \frac{1-\lambda}{2}(p-c) + (1 - F(p|c))\,\lambda\,(p-c) \tag{1}$$

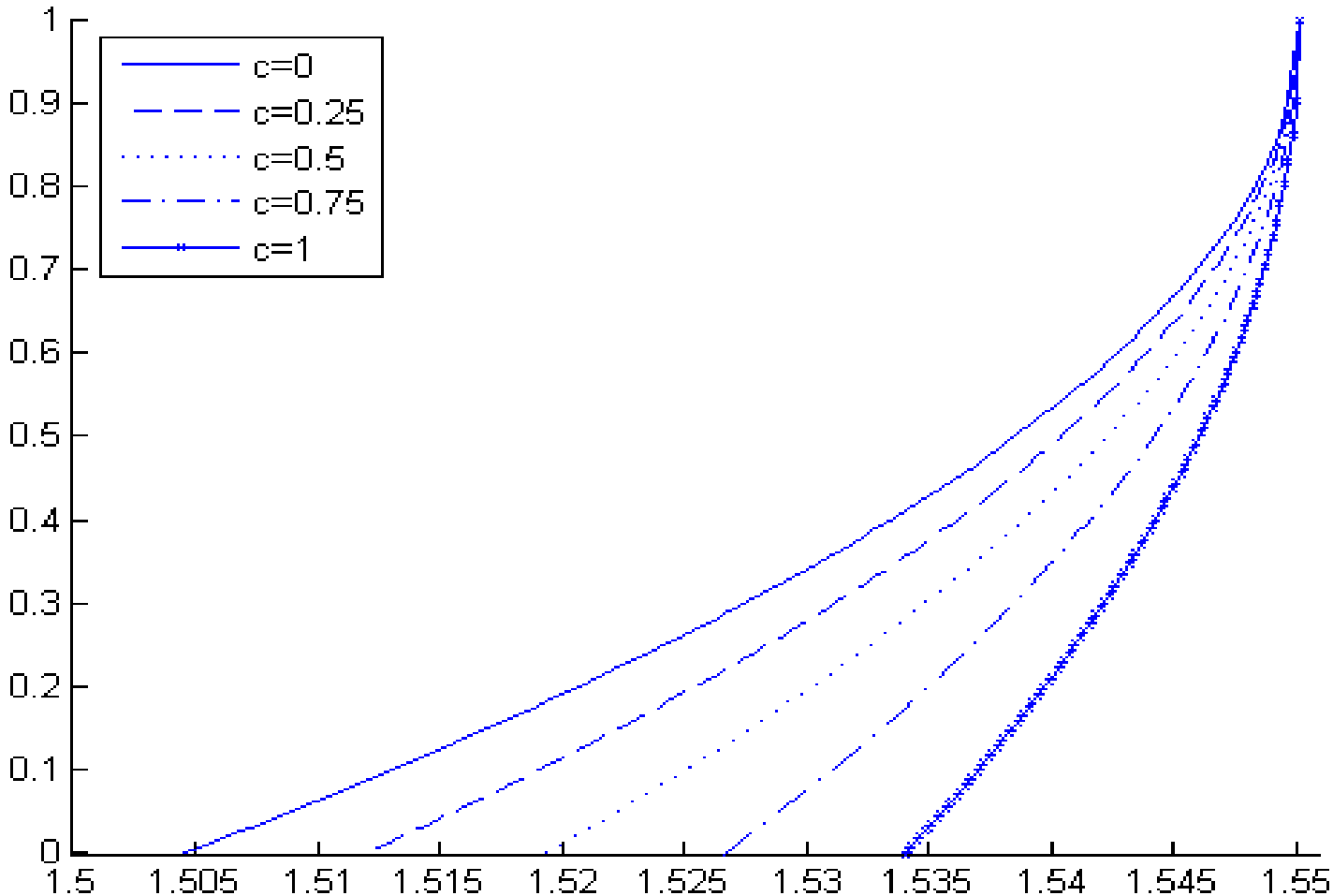
$$= \frac{1-\lambda}{2}(\rho - c) \tag{2}$$

since firms must be indifferent between any price in $p \in \left[\underline{p}(c), \bar{p}(c)\right]$ with $\bar{p}(c) = \rho$.

Consumers'reservation price

$$\rho = s + \Pr(c_L|\rho)E(p|c_L) + \Pr(c_H|\rho)E(p|c_H) \tag{3}$$

Price distributions for different c



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  - We know $E(p|c_L) < E(p|c_H)$
  - If at $p > \rho, \Pr(c_L|p) < \Pr(c_L|\rho)$, then consumer would like to buy at price just above ρ

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  - If at $p > \rho, \Pr(c_L|p) < \Pr(c_L|\rho)$, then consumer would like to buy at price just above ρ
- Thus, reservation price equilibrium requires out-of-equilibrium beliefs to satisfy $\Pr(c_L|p) \leq \Pr(c_L|\rho)$ for $p > \rho$

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  - As RHS is decreasing in c_i for all $p > \rho$, high cost firms have a wider range of responses from the consumers for which it is profitable to deviate
- D1 refinement requires $\Pr(c_L|p) = 0$ for all $p > \rho$. Inconsistent with reservation price equilibrium.

Active Search in Nonreservation Price Equilibria without Mass Point

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- By selling only to nonshoppers, profit of low cost firm reaches a maximum at $p = \overline{p}$: Maximizing $\frac{1-\lambda}{2}\beta(p)(p - c_L)$ gives

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- As $\beta(\overline{p}) > 0$, it follows that $\beta'(\overline{p}) < 0$ and therefore, $\beta(\overline{p}) < 1$.
- Thus, there is an interval of prices where consumers search with strictly positive probability
- Indifferent over interval of prices where $p = s + \Pr(c_L|p)E(p|c_L) + \Pr(c_H|p)E(p|c_H)$

CDF in Nonreservation Price Equilibria without Mass Point

Firms' demand when setting price p and cost is $c_i, i = H, L$ is $D(p|c_i) =$

$$\lambda(1-F_i(p))+\frac{1-\lambda}{2}\beta(p)+\frac{1-\lambda}{2}(1-\beta(p))(1-F_i(p))+\frac{1-\lambda}{2}\int_p^{\overline{p}}(1-\beta(\widetilde{p}))f_i(\widetilde{p})d\widetilde{p} \tag{4}$$

- Profit equals $\pi(p|c_i) = D(p|c_i)(p - c_i) = \frac{1-\lambda}{2}\beta(\overline{p})(\overline{p} - c_i)$.

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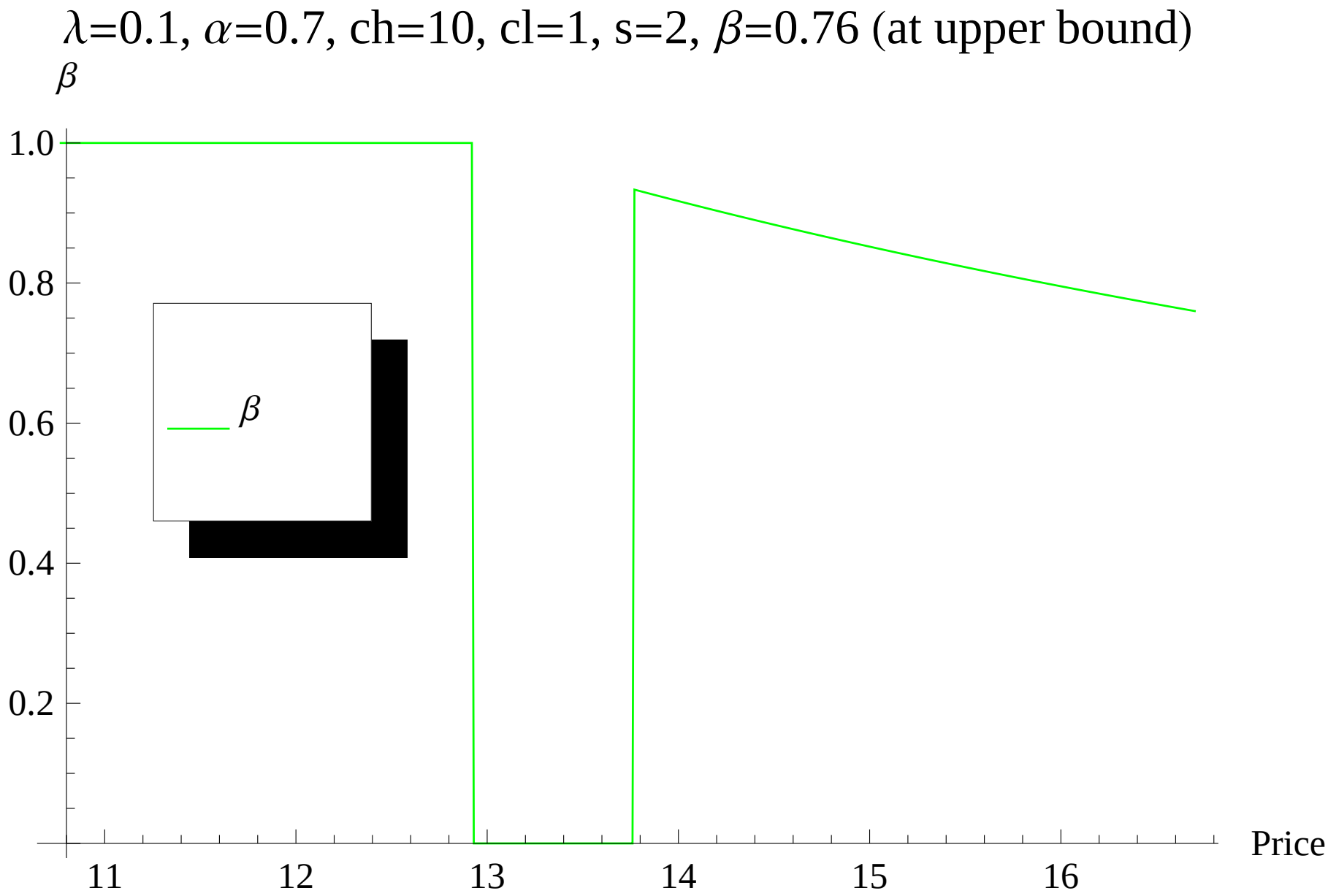
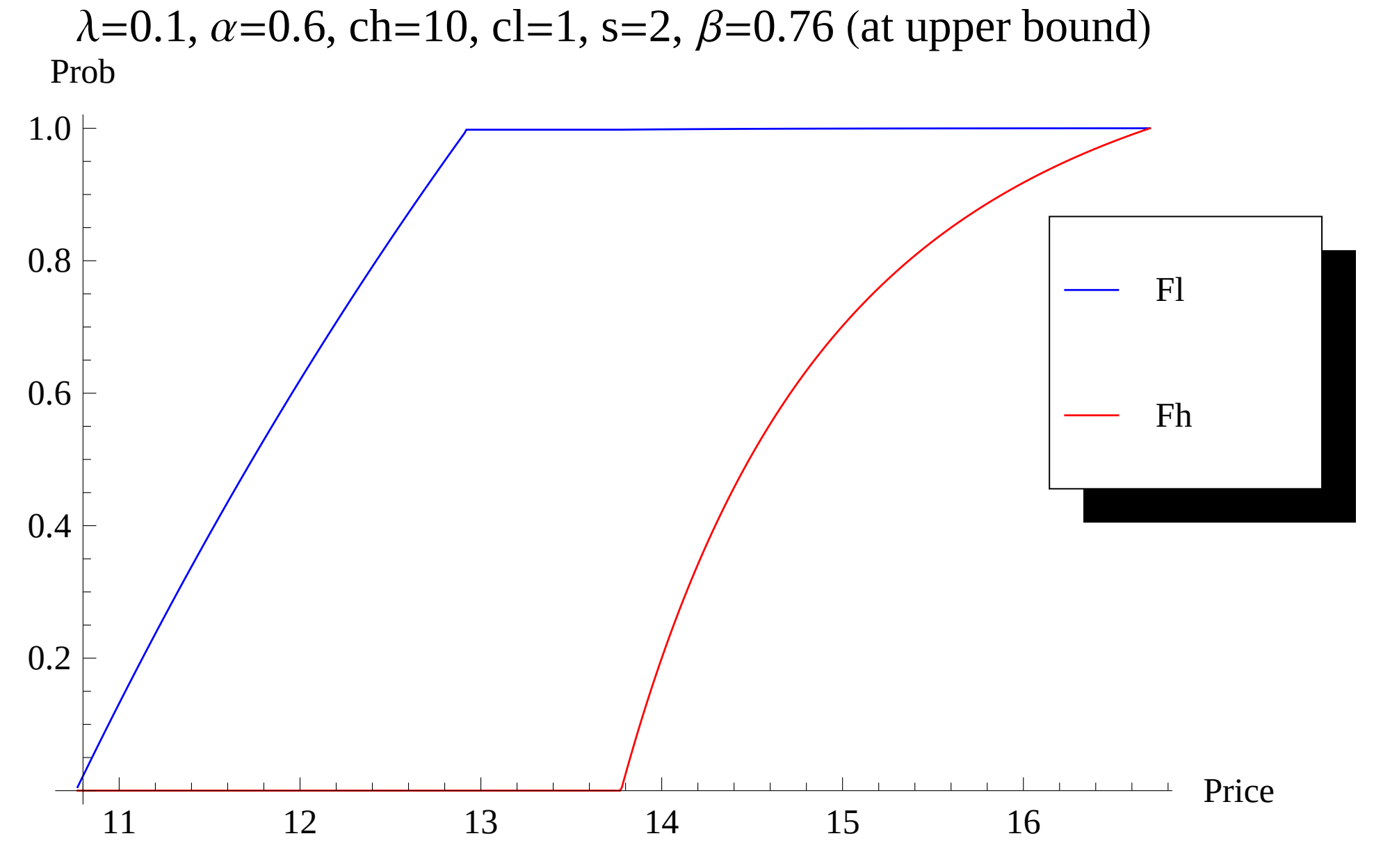
- Profit equals $\pi(p|c_i) = D(p|c_i)(p - c_i) = \frac{1-\lambda}{2}\beta(\overline{p})(\overline{p} - c_i)$.
- By solving an inexact differential equation, equilibrium price distribution in interval where $0 < \beta(p) < 1$ can be solved for

$$F_i(p) = \frac{\int_{\underline{p}_i}^{p} \frac{(1-\lambda)\beta(\overline{p})(\overline{p}-c_i)}{(\widetilde{p}-c_i)^2\sqrt{1-(1-\lambda)\beta(\widetilde{p})}}d\widetilde{p} + C_i}{2\sqrt{1-(1-\lambda)\beta(p)}}. \tag{5}$$


λ=0.1, α=0.7, ch=10, cl=1, s=2, β=0.76 (at upper bound)
β
1.0
0.8
0.6
0.4
0.2
β
11
12
13
14
15
16
Price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11
12
13
14
15
16
Price
Fl
Fh

Nonreservation Price Equilibria with a Mass Point

Consumers

- Buy for sure up to ρ. Search for sure after observing a price $\rho < p < \bar{p}$ and a price $p > \bar{p}$ Buy with probability β at $p = \bar{p}$.

$$\rho = s + \Pr(c_L|\rho)E(p|c_L) + \Pr(c_H|\rho)E(p|c_H) \tag{6}$$

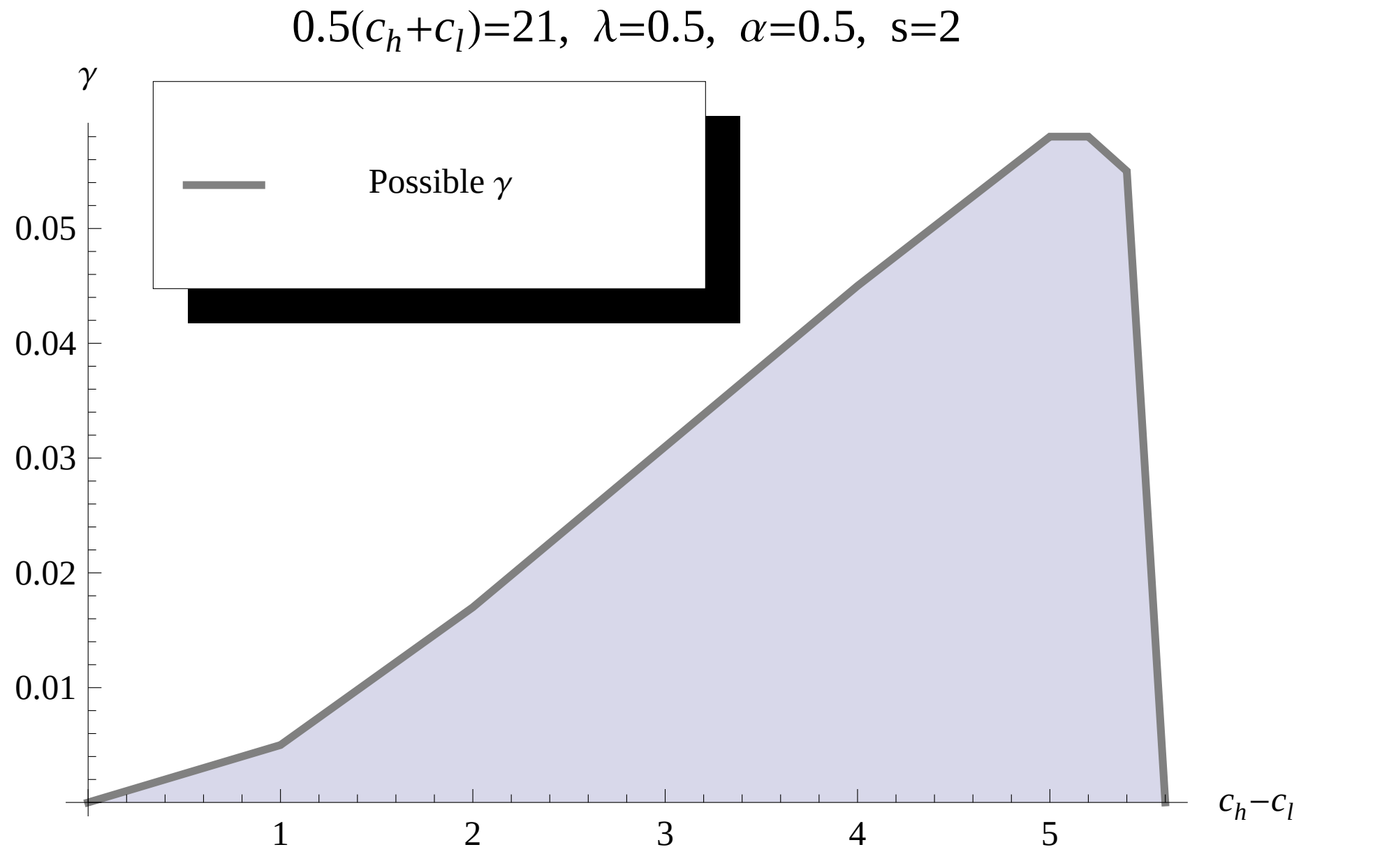
$$\bar{p} = s + E(p|c_H) \tag{7}$$

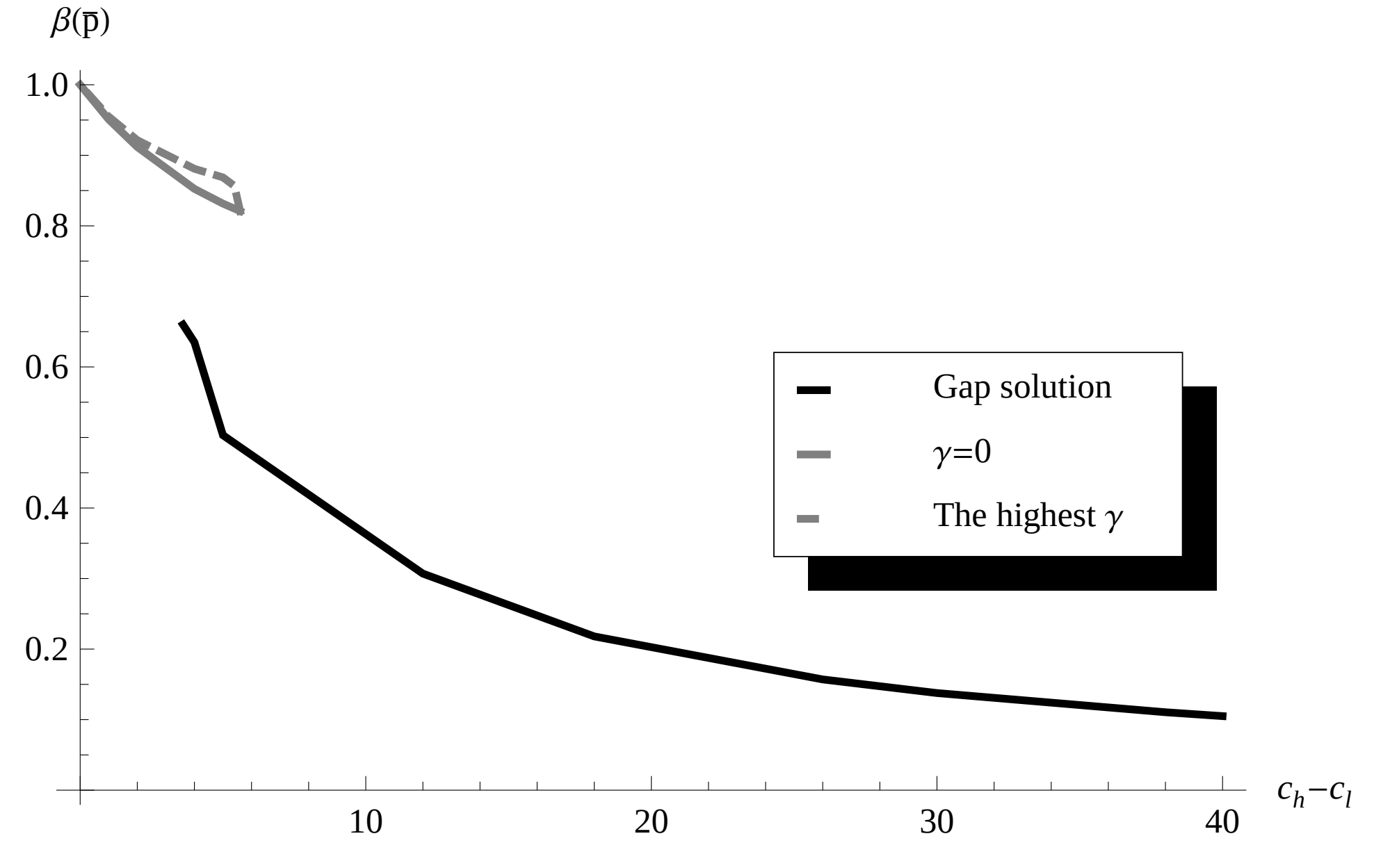
Firms

- Low cost firms set prices at or below ρ
- High cost firms choose $\bar{p}$ with probability γ and set prices at or below ρ with probability $1 - \gamma$.

Nonreservation Price Equilibria with a Mass Point

- At $\rho < p < \bar{p}$ consumers should believe that cost is low and search. This is consistent with D1 if $\beta > \frac{\gamma}{(1-\lambda)[(1-\gamma)(1-x)+\gamma]}$, where $x > 1$ is a number measuring how much larger is the equilibrium profit of a low cost firm compared to if it would set $\bar{p}$.
- Undercutting is not optimal given these beliefs if *beta* is large enough.
- For any $\gamma > 0$ there is active search in this equilibrium with positive probability.
- For γ converging to 0, equilibrium effectively converges to equilibrium in Dana (1994) and generalized in Janssen et al. (2011)


0.5(c_h+c_l)=21, λ=0.5, α=0.5, s=2
γ
Possible γ
0.05
0.04
0.03
0.02
0.01
1
2
3
4
5
c_h−c_l


$\beta(\bar{p})$
1.0
0.8
0.6
0.4
0.2
10
20
30
40
$c_h - c_l$
Gap solution
γ=0
The highest γ

Existence of Nonreservation Price Equilibria?

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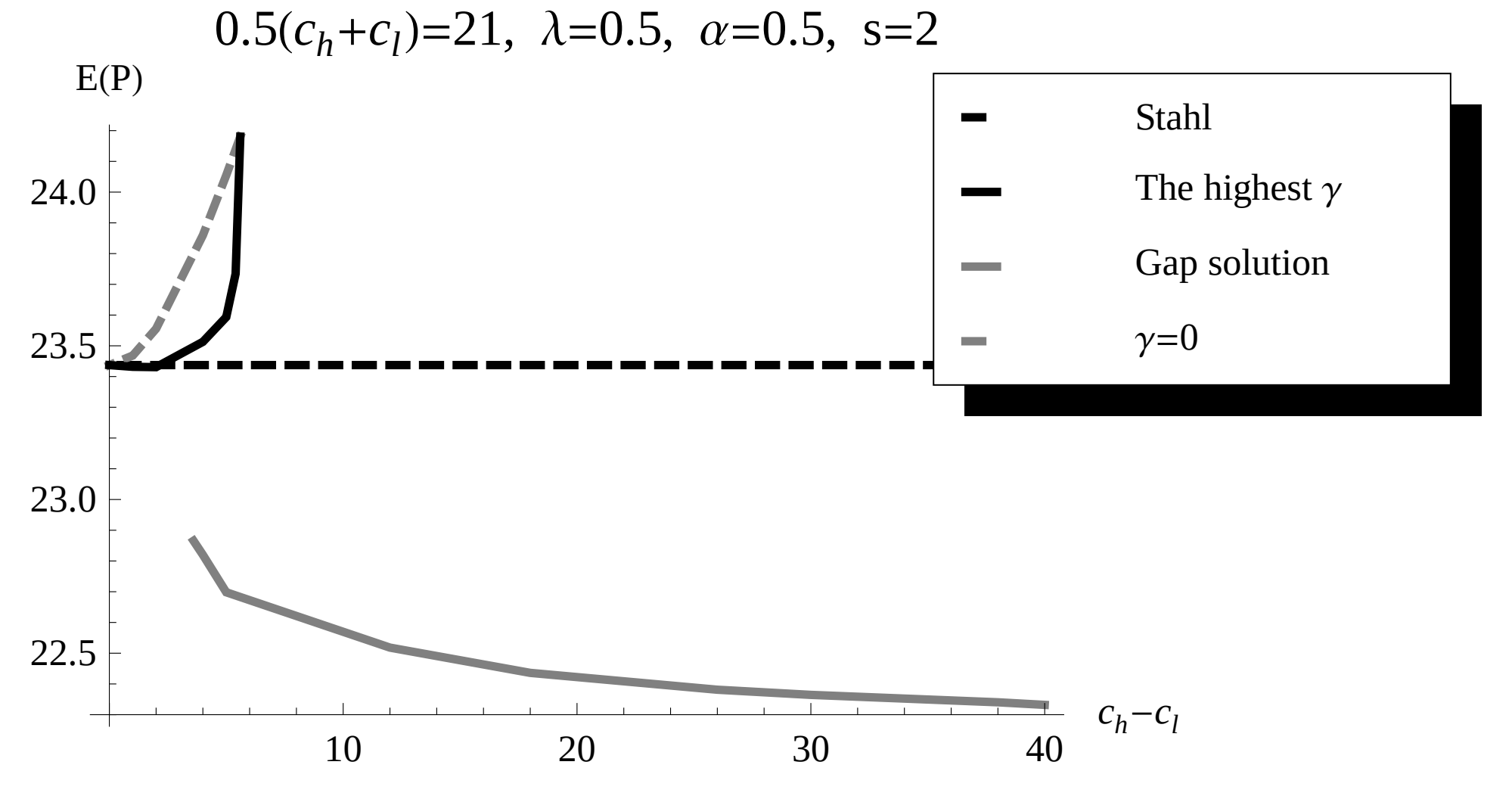
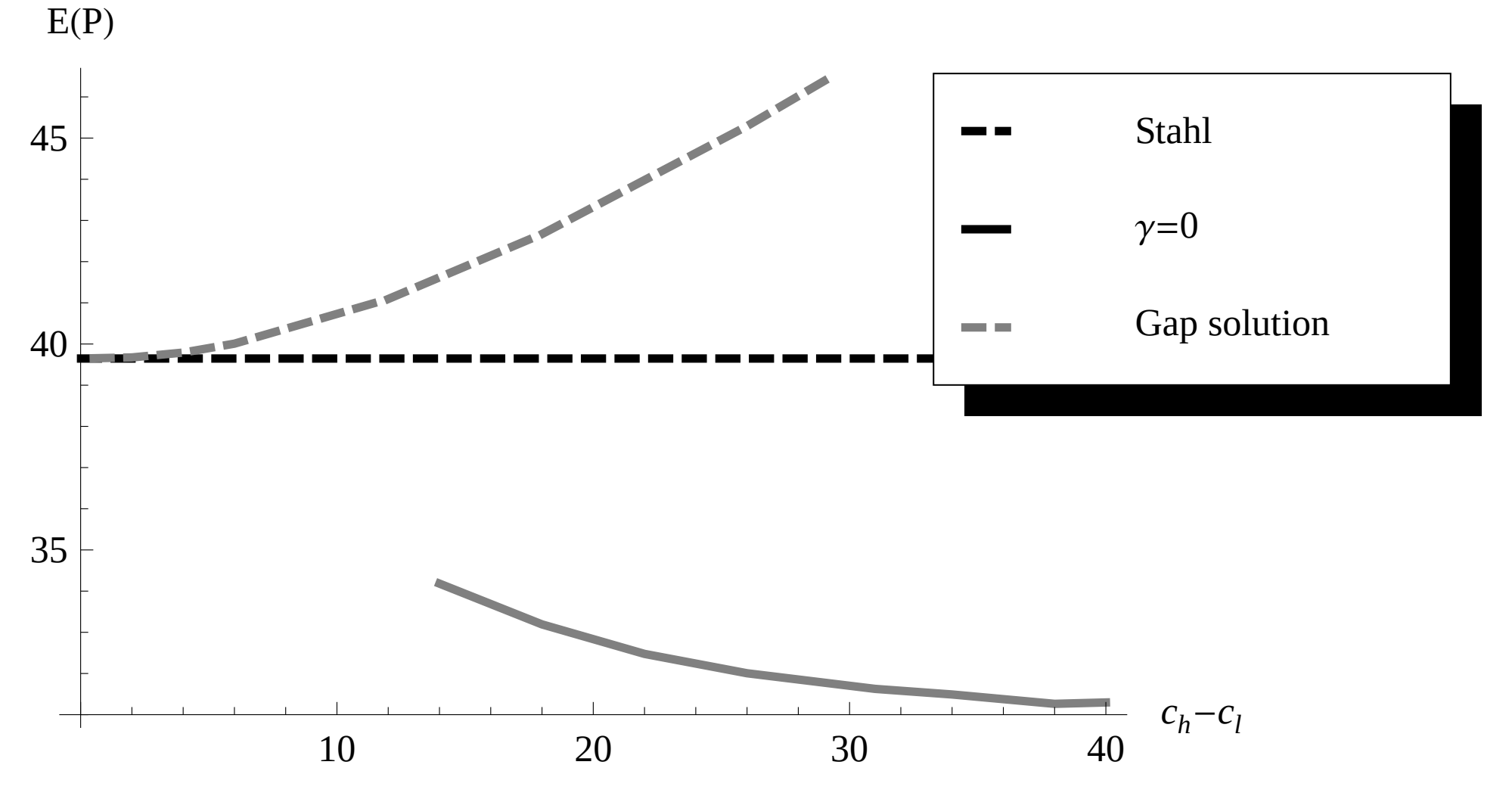
- Equilibrium with mass point does not exist for large cost differences
- Equilibrium without mass point does not exist for low cost differences
  - For this equilibrium to exist we should have that at $\underline{p}(H)$ $\beta(p) < 1$.
  - In other words, nonshoppers should randomize over the whole interval of prices set by high cost firms. And there should be a gap in the low cost distribution.
- Simulation results suggest either one or the other or both types of equilibria exist for any choice of parameter values.


0.5(c_h+c_l)=21, λ=0.5, α=0.5, s=2
E(P)
24.0
23.5
23.0
22.5
10
20
30
40
c_h−c_l
Stahl
The highest γ
Gap solution
γ=0


E(P)
45
40
35
10
20
30
40
$c_h - c_l$
Stahl
γ=0
Gap solution

Conclusion

- If consumers do not know prices, it is likely they also do not know firms' cost.
- When consumers are uncertain about firms' cost, reservation price strategy is unreasonable
- Forces us to inquire about equilibria where consumers do not have a reservation price
- We show these equilibria always have active search along equilibrium path
- "Show" they always exist. Work in progress
- Equilibrium price distributions very different from previous characterizations. Some "parametric" freedom. New challenge for people doing empirical work.
- Numerical simulations suggest that expected prices are lower than those in reservation price equilibria and sometimes even lower than in case cost is known