Equilibrium Long-Tailed Sales in a Search Model

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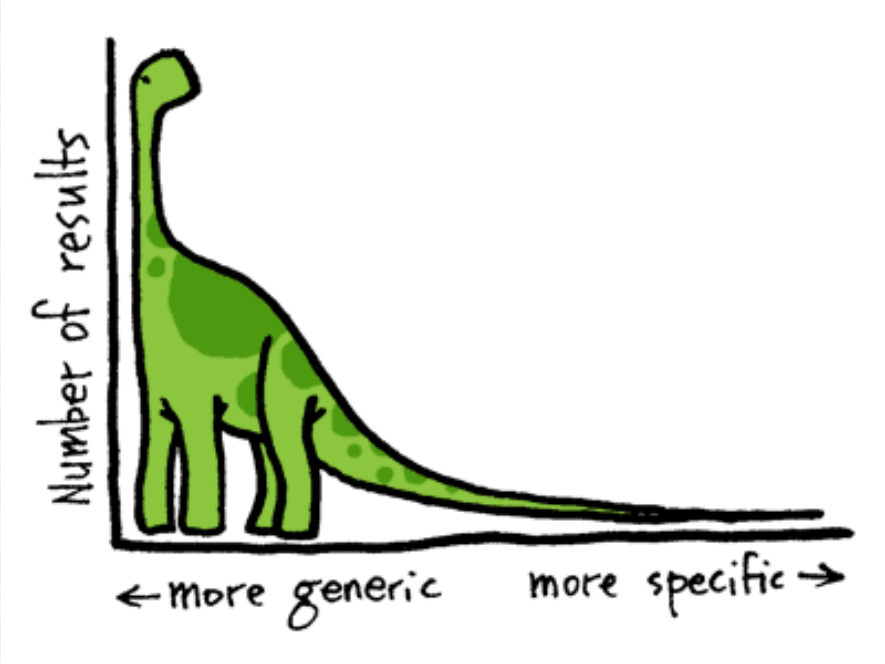
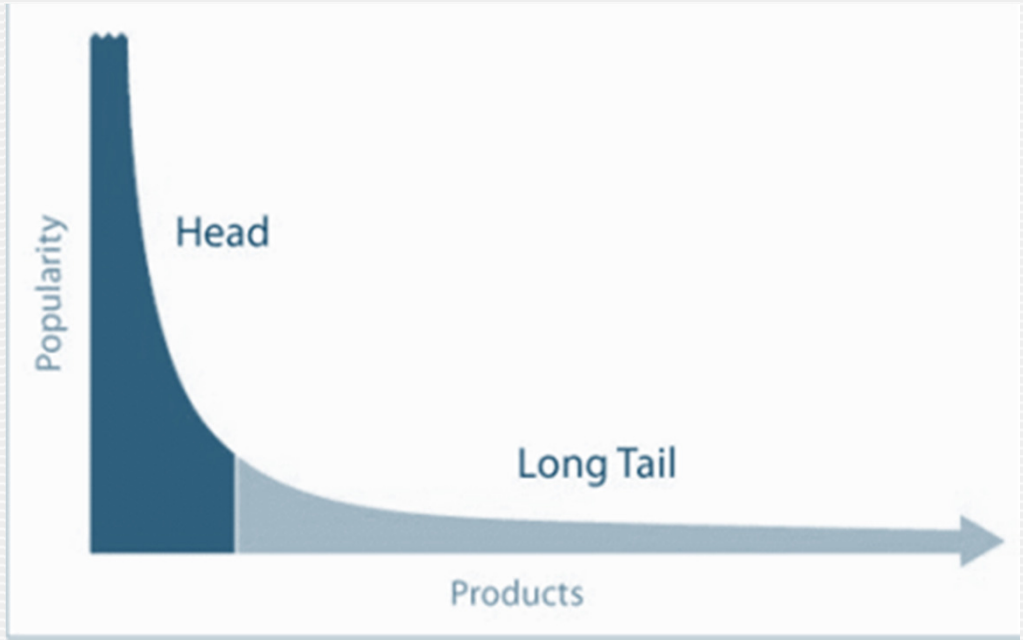
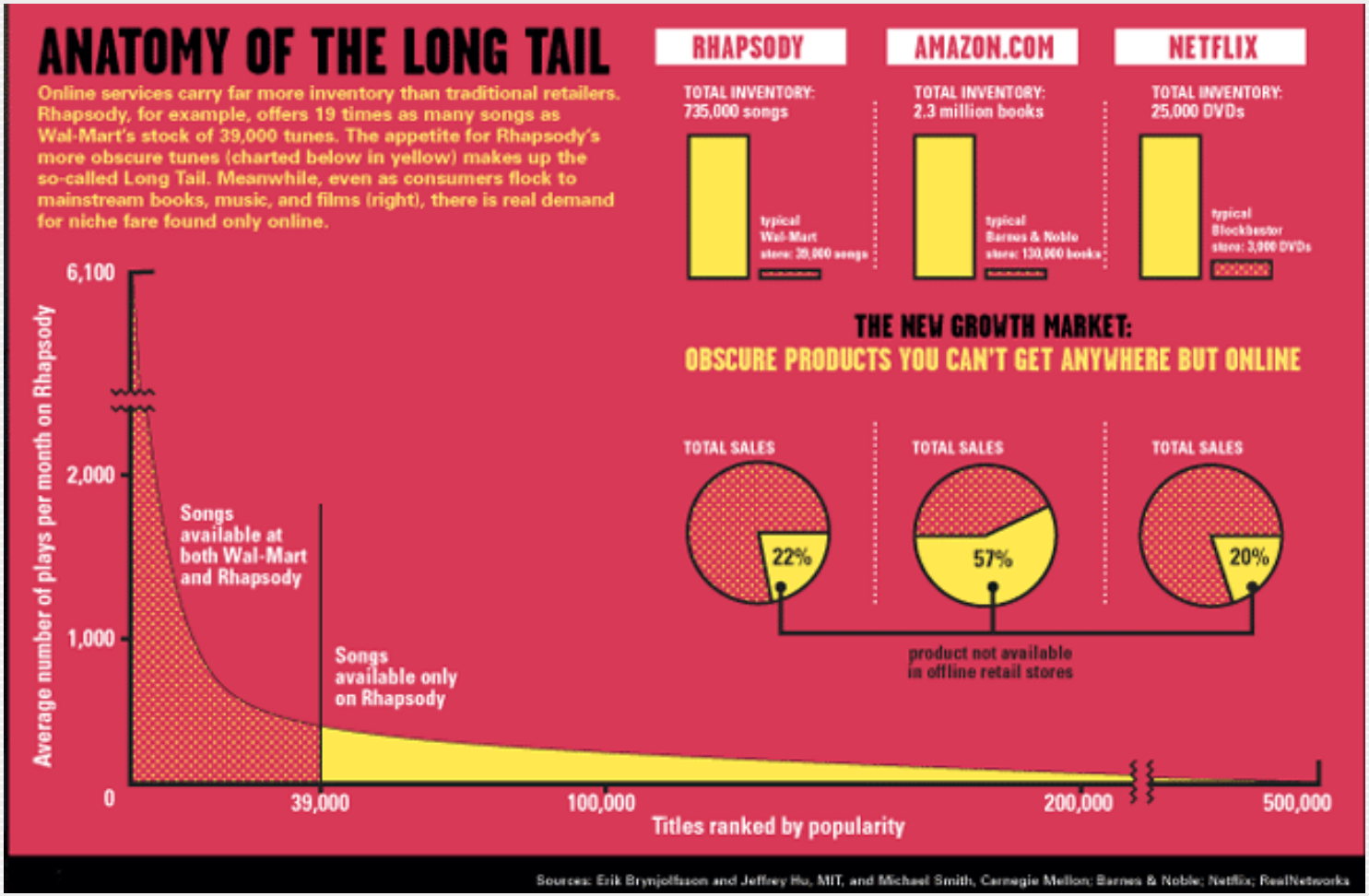


What is the long tail phenomenon?

Long tailed markets are markets where the aggregate sales of obscure or niche products constitute a significant portion of total sales.



Motivation: The Long Tail is a Significant Phenomenon



Motivation: Why is the long tail interesting?

- Long tails in the online book, music and video industries (Brynjolfsson, Hu and Smith, 2006; Anderson, 2006).
- A change of business models → market of niches.
- Amazon's tail (Brynjolfsson, Hu and Smith 2010):
  - Books outside the top 100,000 accounted for 36.7% total sales in 2008.
  - Consumer surplus generated by the availability of niche books is between $3.9-$5 billion.

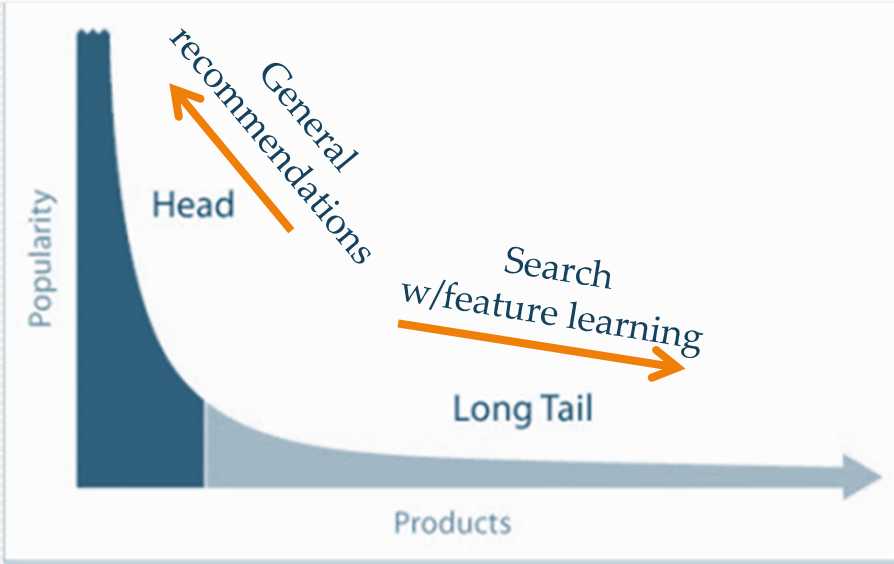
The long tail is a significant phenomenon, with implications for both firm profits and consumer welfare.

Related Literature Explaining the Long Tail

- Supply-side explanations do not tell the whole story (Elberse and Oberholzer-Gee 2008; Brynjolfsson, Hu and Simester, 2011)
- Recommendation systems → longer tail + lower sales concentration (Oestreicher-Singer and Sundararajan 2009; Brynjolfsson, Hu and Simester, 2011).
- Effects of online reviews (Chevalier and Mayzlin, 2006; Ehrman and Schmale, 2008; Duan and Zhou, 2009).
- Bar-Isaac, Caruana and Cuñat (*AER* 2011) – endogenous product design.
- Recent working papers: Hervas-Drane (2010), Yang (2011) – LT driven by heterogeneous preferences.

This work: A theoretical demand side explanation based on consumers' search strategies

"Infinite" supply (online) + Effective search (online)



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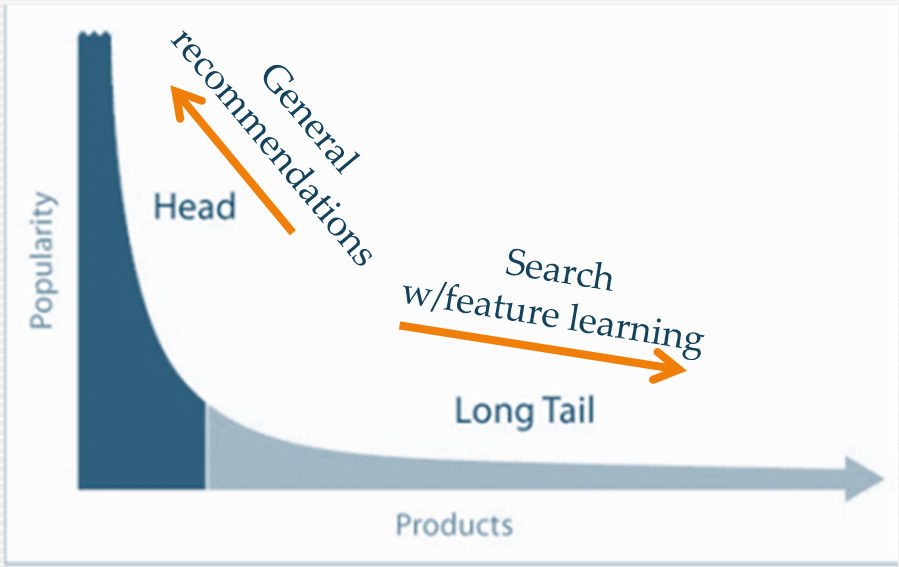

This work provides:

- A demand side explanation to the Long Tail, in a search framework.
- Conditions for a market equilibrium with long tailed sales.
- The functional form for the equilibrium sales distribution as a function of consumers' search strategies.

Outline of the Model

Firms and the sales distribution:

- N firms, one product each, $MC = 0$.
- Firm i sets price p_i.
- σ_i - firm i's market share, f - the sales distribution (determined endogenously in equilibrium).
- Firm i set price p_i given σ_i, f, and consumers' search and purchase behavior.

Consumers:

- K consumers. Unit demand.
- Consumers know the general form of the sales distribution, f.
- Cannot learn market shares without conducting search.
- First choose a search strategy, then make a purchase decision.

The model: Consumers

- K homogeneous consumers: $k \in \{1..K\}$
- Consumers know f.

Consumer utility:

$$u_{i,k} = v_{i,k} - p_i - c_k$$

Match value — Price — Search cost

The model: Consumers

Consumer utility: $u_{i,k} = v_{i,k} - p_i - c_k$

Match value:

$$v_{i,k} = \begin{cases} v_H & w.p.\ \frac{1}{N} \quad \text{Perfect match} \\ v_L & w.p.\ \sigma_i(1-\frac{1}{N}) \quad \text{Good match} \\ 0 & w.p.(1-\sigma_i)(1-\frac{1}{N}) \quad \text{No match} \end{cases}$$

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- One perfect match per consumer, independent across consumers and products.
- σ_i gives the probability that i is a good match for all.
- Correlation w.r.t. good matches but not for perfect matches.
- f represents the distribution of the "general taste".
- (Good and perfect matches are not mutually exclusive).

The model: Consumers' Search and Purchase Behavior

Each consumer chooses one of two search procedures:

Sampling a recommendation:

Searching the online aggregator:

The model: Consumers' Search and Purchase Behavior

Each consumer chooses one of two search procedures:

Sampling a recommendation:

- Product i is sampled w.p. σ_i.
- Consumer learns $\sigma_i, p_i, Ev_i \equiv E\left[v_{i,k} \,|\sigma_i\right]$

$$Ev_i = \frac{1}{N} v_H + \left(1 - \frac{1}{N}\right) \sigma_i v_L$$

- Free: $c_k^r \equiv 0$
- Purchase decision: $Ev_i - p_i \geq 0 \rightarrow$ buy

The model: Consumers' Search and Purchase Behavior

Each consumer chooses one of two search procedures:

Searching the online aggregator:

- Full product info.
- Consumer k learns $v_{i,k}, p_i \ \ \forall i,k.$
- Cost: $c_k^a \sim C^a[\underline{c}, \bar{c}]$. C^a known to firms.
- Purchase decision:

  Buy i^*: $\ i^* = argmax_i(v_{i,k} - p_i)$

- Only if non-neg.

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- Only if non-neg.

$$E\left[\max\left(v_{i,k} - p_i\right)\right] \geq c_k^a \rightarrow a, \text{ otherwise} \rightarrow r$$

The model: Consumer and Firm Strategies

Consumers:

Search strategy: $S(f, c_k^a) \in \{r, a\}$

(purchase rule corresponds to search strategy)

Firms:

Pricing: $P(\sigma_i, f, S, C^a) = p_i \in [0, \infty)$

We proceed to define market equilibrium

The model: Market Equilibrium

Definition 3: Market equilibrium is a triplet $\langle f, P, S \rangle$ such that:

1. $P(\sigma_i, f, S, C^a)$ is optimal given f and S (and the corresponding purchase rule).
2. $S(f, c_k^a)$ is optimal given f and P.
3. When agents employ equilibrium strategies P and S, the sales distribution is f.

A Comment on Stability

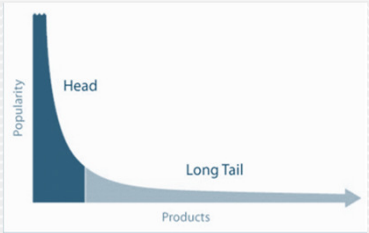
A pseudo dynamic interpretation of the model:

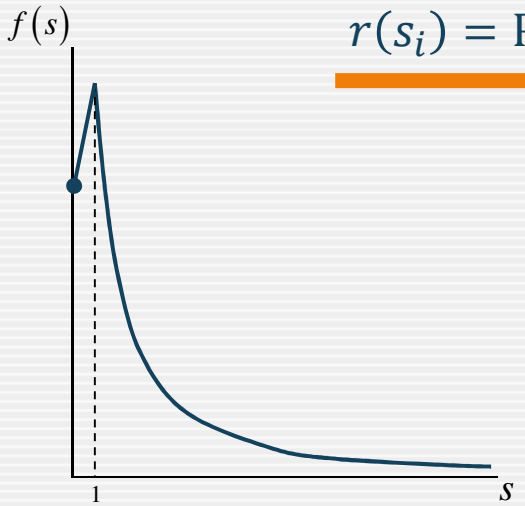
- A new group of consumers enters and makes purchases in every period.
- f_t is the distribution of all sales accumulated up to the beginning of period t.
- f_{t+1} is determined by f_t and S.

Definition 4: An equilibrium in the dynamic setting is stable when $f_t \to f^*$.

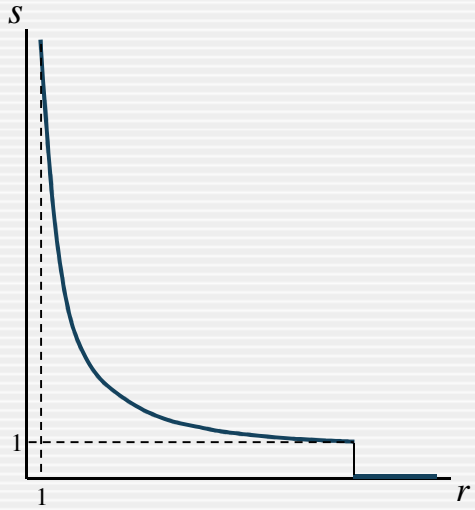
Defining the Long Tail in the Model

- The tail of interest is the tail of the sales-rank distribution.





$$r(s_i) = \Pr[s \geq s_i] \cdot N$$



The sales distribution

The sales-rank "distribution"

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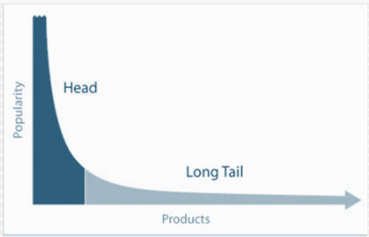

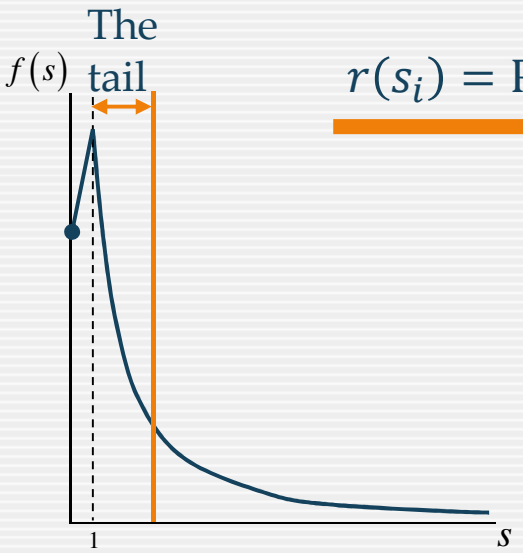

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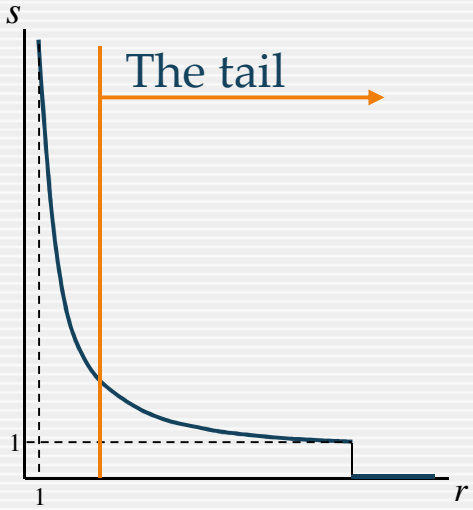


The sales distribution

The sales-rank "distribution"

Equilibrium: Firm pricing

Plan: equilibrium pricing → search strategies → f

Lemma 1: When N is large and v_H is large enough*, the equilibrium pricing strategy:

$$p_i = Ev_i$$

Intuition:

- One perfect match at aggregator: probability $\frac{1}{N} \to 0$ → doesn't affect pricing.
- Recommendation searchers see monopolistic prices at WTP.
- Assumption *: $v_H - \max_i\{Ev_i\} \geq v_L$ → perfect match always chosen at aggregator.

Equilibrium: Consumers' Search and Purchase Behavior

Definition 5: An α-search behavior is the use of the following heuristic rule:

1. With probability α draw one product from $\{1..N\}$ uniformly at random, and purchase it.
2. With probability $(1-\alpha)$ copy a previous purchase: sample one product from $\{1..N\}$ where product i is sampled with probability σ_i, and purchase it.

Proposition 1: Consumers' optimal search and purchase behavior is equivalent to an α-search behavior with $\alpha = \Pr[c_k^a \leq v_H - Ep_i]$.

- Recommendation-based search: i sampled w.p. σ_i and purchased w.p. 1.
- At the aggregator: each product is chosen w.p. $\frac{1}{N}$

Equilibrium f: α-Search Behavior Implies Equilibrium Long Tail

Lemma 2: When consumers' equilibrium behavior is equivalent to an α-search behavior then a distribution f^* which follows a power law for low levels of s is an equilibrium distribution, and its corresponding sales-rank distribution has a power law tail.

Specifically: $f^*(s) = \Theta\left(s^{-\frac{2-\alpha}{1-\alpha}}\right)$ for $1 \leq s \leq \log N$, and $f^*(0) = \frac{1}{1+\alpha\bar{S}/N}$

$$s^*(r) = \Theta\left(r^{-(1-\alpha)}\right)$$

f^* is the limiting sales distribution when the market is constructed by an α process (adaptation of Yule, 1925; strong convergence result by Kumar et al., 2000).

Intuition – next slide

Proposition 2: When $N \to \infty$ there exists an equilibrium with long tailed sales. The tail of the equilibrium sales rank distribution is a power law with exponent $(1-\alpha)$, where $\alpha = \Pr[c_k^a \leq v_H - Ep_i]$. Furthermore, the equilibrium is stable.

Equilibrium Long Tail - Intuition

α-search behavior:

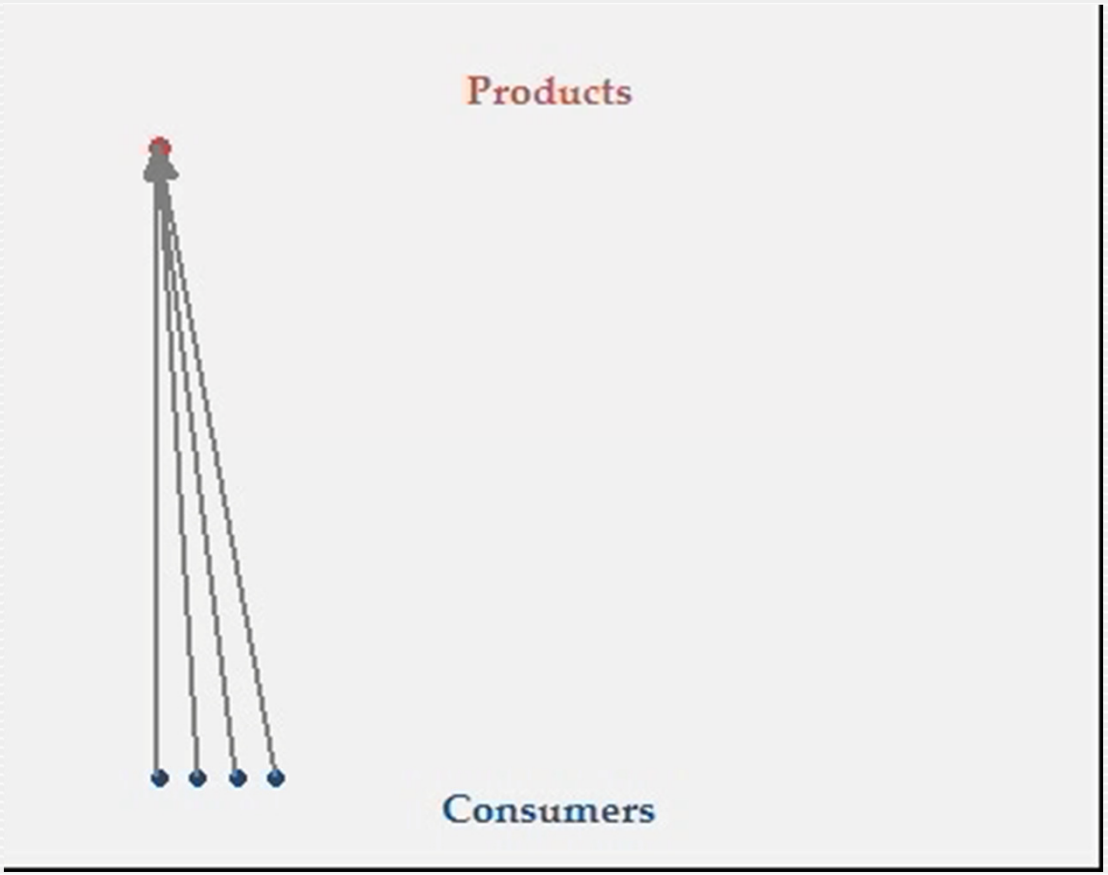
- Copying - preferential attachment mechanism.
- Randomizing - allows unpopular products to gain popularity.

The result: an equilibrium market where most products have low sales, and are considered "tail" or "niche", and a few products are best sellers, or "hits".

When N is large:

- Consumers find it optimal to act *as if* they are using the α heuristic.
- The resulting market structure is long tailed.
- This result is stable.

Equilibrium Long Tail - Intuition



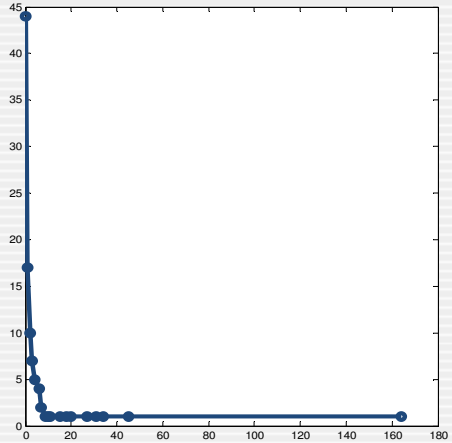
http://www.youtube.com/watch?v=54XxcDqe8Hg

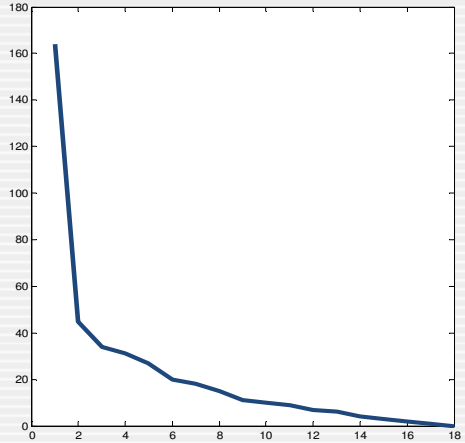
Equilibrium Long Tail - Simulations

The sales distribution - $f(s)$

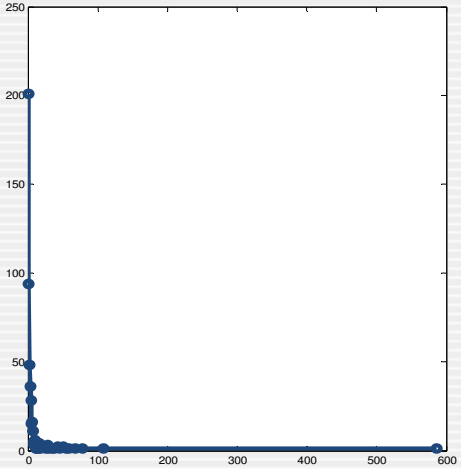
The sales-rank distribution - $s(r)$

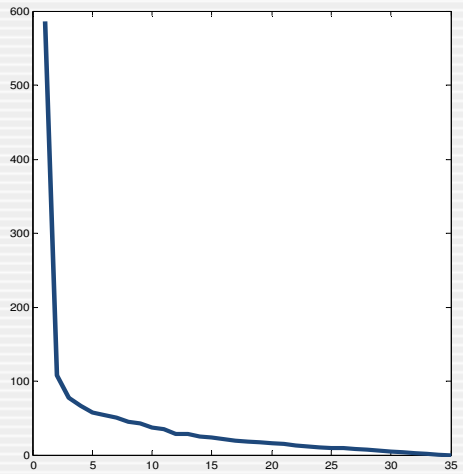
$N = 100$:





$N = 500$:





Equilibrium Long Tail - Simulations

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The sales-rank distribution - $s(r)$

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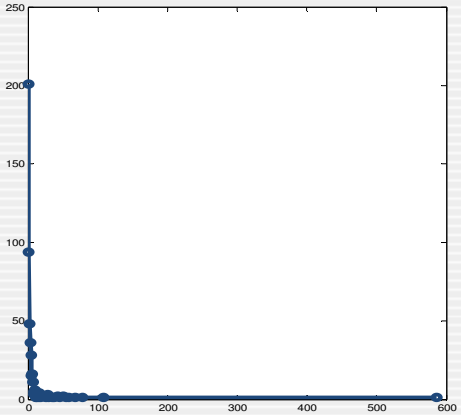
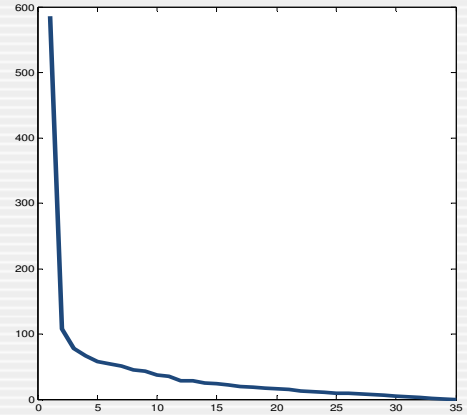
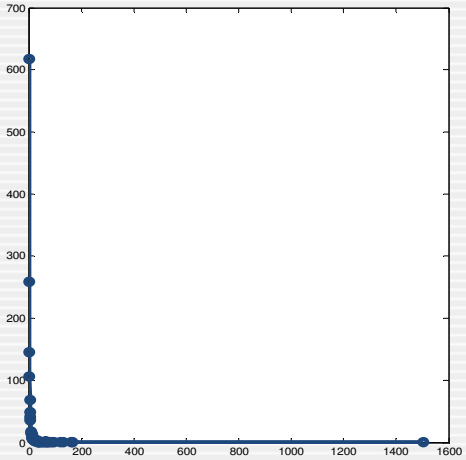
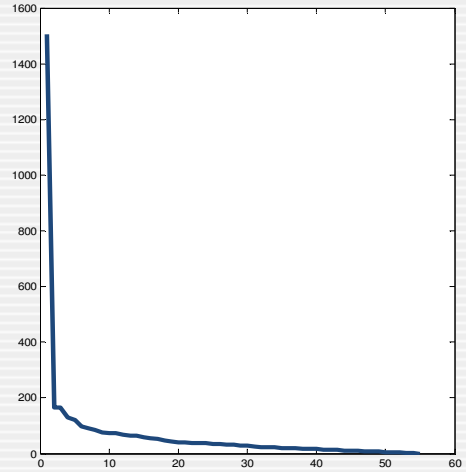




$N = 1500$:

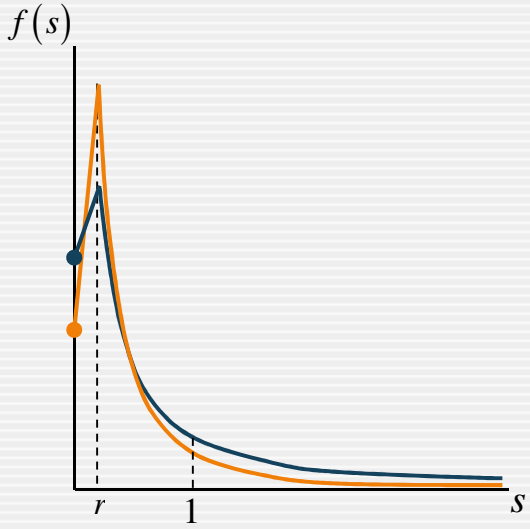




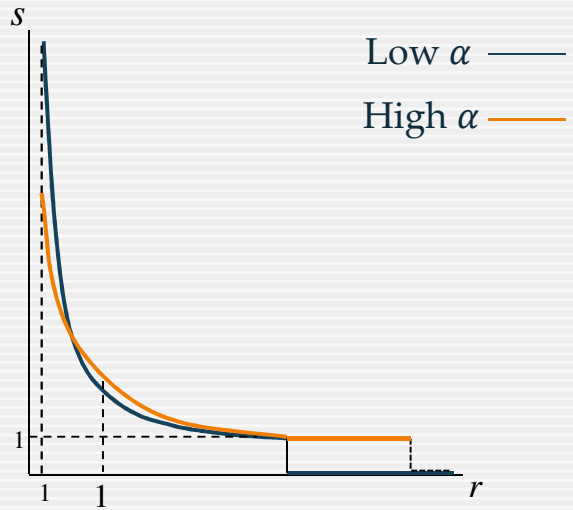
Decreases in the Online Search Cost Lead to a Longer and Heavier Tail

Proposition 3: Increases in α result in a shift to a new equilibrium sales and sales-rank distributions with a longer and heavier tail.

- Recall: $\alpha = \Pr[c_k^a \leq v_H - Ep_i]$
- $\alpha \uparrow$ represent changes in C^a due to better or cheaper online search.



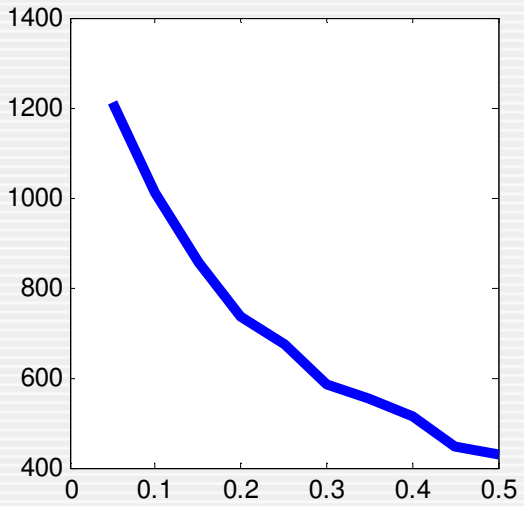
The sales distribution



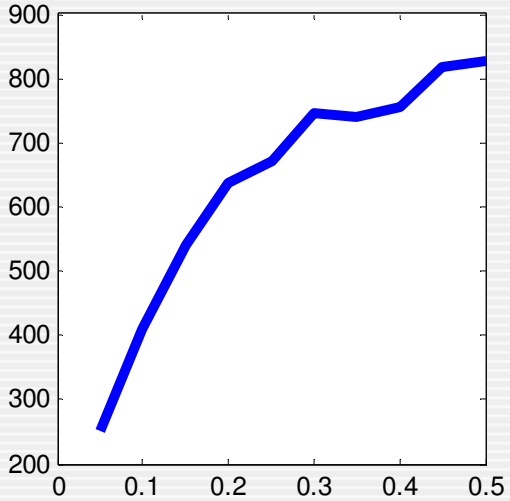
The sales-rank distribution

Decreases in the Online Search Cost Lead to a Longer and Heavier Tail - simulations

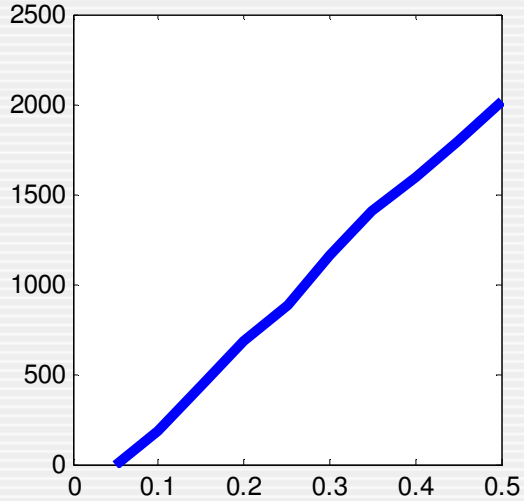
Number of 0-sellers



Length of the tail
$(1 \leq s \leq 7)$



Weight of the tail
$(0.8N$ worst sellers)



α: 0.1→0.2 → weight increases by 260%.

For $N = 1500$, $\bar{S} = 7500$

Conclusion

- A demand side explanation of the long tail phenomenon, based on consumers' use of both traditional and web-based search procedures.
- Main result: the existence of an equilibrium with long tailed sales.
- In line with empirical evidence of long tailed sales-rank distributions in online markets.
- As online search tools become "cheaper" or easier to use, the tail of the market becomes longer and heavier.

The End