Search with Adverse Selection

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June 2012

Introduction–Search with Adverse Selection

- We present a model of search with adverse selection and use it mainly to inquire about information aggregation.

- For a common values auction environment, Wilson and Milgrom derived conditions on the informativeness of the signals under which the price aggregates information when the number of bidders is large.

- We inquire about information aggregation by the price in a search version of this environment. That is, the "auctioneer" samples bidders sequentially and the counterpart of many bidders is small sampling cost.

- We claim that the price aggregates information more poorly in the search environment. That is, for the same signal structure, the price formed by the auction always weakly and sometimes strictly aggregates more information than the price arising in the search.

- This is explained by a stronger winner's curse afflicting the search.

Model

- Buyer samples sequentially from **continuum** of sellers (say, for a service).
- Buyer's value of the transaction is u.
- sampling cost $s > 0$
- Buyer's type $w \in \{L, H\}$. Prior$(w) = \rho_w$
- A seller's cost of providing the service c_w. $c_H > c_L$.

Thus, L is the "good" type.

Model–meeting

- **Signal**
  - On meeting buyer, seller obtains a signal $x \in X = [\underline{x}, \overline{x}]$
  - x's distribution F_w, $w = L, H$, continuous density f_w
  - increasing f_H / f_L: lower x's more indicative of low cost.
  - Conditional on w, signal is independent across sellers.

- **Price**
  - After x observed, nature draws a price p from distribution G over $[0, u]$ (G has full support, continuously differentialble, strictly positive density g).
  - Seller announces whether accepts p
  - Then buyer annouces
  - If both accept, transact at p
  - if not, buyer continues search

Model-payoffs and strategies

- **Payoffs**
  - Buyer's from transacting at price p after sampling n sellers:

$$u - p - ns$$

  where u is sufficiently larger than $c_H + s$.

  - Seller's:

$$\begin{cases} p - c_w & \text{if} \quad \text{transacts} \\ 0 & \text{otherwise} \end{cases}$$

- **Histories**
  - Seller: signal and price. Nothing else.
  - Buyer: entire history of search (does'nt matter whether observes x's).

- **Strategies**
  - Buyer's: $B = (B_L, B_H)$
    where $B_w(\varphi) \subset [0, u]$ are prices accepted by $w = L, H$ after history φ.

  - Seller j's: $A_j(x) \subset [0, u]$ set of accepted prices after signal x.

Model-Symmetric situation (B,A)

- Symmetry/Markov (non-assumption)
  - All sellers use same A
  - B depends only on present price draw.
- Symmetry/markov above **can be derived** rather than assumed.

- $\Omega_w(B, A) =$ price-signal pairs that lead to trade given $B, A, w = L, H$

$$\Omega_w(B, A) = \{(p, x) : p \in B_w \cap A_w(x)\}.$$

- F_w & G induce a probability measure Π_w over sets of (p, x) pairs

$$
\begin{aligned}
\Pi_w(\Omega_w(B, A)) &= \Pr[\text{buyer } w \text{ stops in a given round}] \\
&= \int_{\underline{x}}^{\overline{x}} \int_{p \in B_w \cap A(x)} g(p) f_w(x) \, dp dx
\end{aligned}
$$

- Expected search duration till trade for buyer type $w = L, H$.

$$n_w(B, A) = \frac{1}{\Pi_w(\Omega_w(B, A))}$$

- Expected price paid by buyer type $w = L, H$,

$$p_w(B, A) = E_{(x,p)}\left[p | \Omega_w(B, A), w\right].$$

- Expected payoffs of buyer type $w = L, H$,

$$V_w(B, A) = u - p_w(B, A) - \frac{s}{\Pi_w(\Omega_w(B, A))}.$$

- Seller's INTERIM (after x before p) belief that $w = H$

$$\beta_I(x, B, A) = \frac{\rho_H f_H(x) n_H(B, A)}{\rho_H f_H(x) n_H(B, A) + \rho_L f_L(x) n_L(B, A)}$$
$$= \frac{1}{1 + \frac{\rho_L}{\rho_H} \frac{f_L(x)}{f_H(x)} \frac{n_L(B, A)}{n_H(B, A)}}$$

- 3 LR's: prior LR $= \frac{\rho_L}{\rho_H}$, signal LR $= \frac{f_L(x)}{f_H(x)}$, sampling LR $= \frac{n_L(B, A)}{n_H(B, A)}$

- β_I depends on (B, A) only via $\frac{n_L(B, A)}{n_H(B, A)}$.

Model-Interim Beliefs

- Assume: finite $\#$ of sellers N; buyer samples uniformly.
- $\mu_w = \Pr\{\text{disagreement in buyer } w- \text{ seller encounter}\}$
- $\Pr\{\text{seller is sampled}|\ w\}$

$$= \tfrac{1}{N} + \tfrac{N-1}{N}\mu_w \tfrac{1}{N-1} + \tfrac{N-1}{N}\tfrac{N-2}{N-1}\mu_w^2 \tfrac{1}{N-2} + ... + \tfrac{N-1}{N}\tfrac{N-2}{N-1}\cdots\tfrac{1}{2}\mu_w^{N-1}$$

$$= \tfrac{1}{N}(1 + \mu_w + \mu_w^2 + ... + \mu_w^{N-1}) = \tfrac{1}{N}\tfrac{1-\mu_w^N}{1-\mu_w}$$

- $\Pr\{H\ |\text{seller is sampled}, x\}$

$$= \frac{\rho_H f_H(x)\frac{1-\mu_H^N}{1-\mu_H}}{\rho_H f_H(x)\frac{1-\mu_H^N}{1-\mu_H}+\rho_L f_L(x)\frac{1-\mu_L^N}{1-\mu_L}} \xrightarrow[N\to\infty]{} \frac{\rho_H f_H(x)n_H(P,A)}{\rho_H f_H(x)n_H(P,A)+\rho_L f_L(x)n_L(P,A)} \equiv \beta_I.$$

- Infinite seller case: continuum–conditioning on 0-prob event of seller being sampled (if instead countable–improper prior).

Model-equilibrium

EQUILIBRIUM: strategies $B = (B_L, B_H)$, A and beliefs
$\beta(x, p) = \Pr(H \mid x, p$ is accepted by buyer$)$ s.t.

(i) B_w maximizes $V_w(B, A)$, $w = L, H$:

$$p \in B_w \text{ iff } u - p \geq V_w(B, A)$$

(ii) $A(x)$ maximizes seller expected payoff given beliefs $\beta(x, p)$ over state:

$$p \in A(x) \text{ iff } p \geq \beta(p, x) c_H + (1 - \beta(p, x)) c_L.$$

(iii) $\beta(p, x)$ is consistent with B, A and distributions $F \& G$.

$$\beta(p, x) = \frac{\beta_I(x; B, A) 1_{p \in B_H}}{\beta_I(x; B, A) 1_{p \in B_H} + [1 - \beta_I(x; B, A)] 1_{p \in B_L}}.$$

Equilibrium – Existence & Characterization

- An equilibrium exists.

- **Result:** If (B, A) an equilibrium, then $V_L(B, A) > V_H(B, A)$

- Let $E_I[c|x, B, A]$ denote INTERIM cost

$$E_I[c|x, B, A] = \beta_I(x|B, A)c_H + [1 - \beta_I(x|B, A)]c_L$$

- Suppress arguments (B, A) and write Ω_w, n_w, V_w, $\beta_I(x)$ and $E_I[c|x]$.

- In equilibrium

$$B_w = [0, u - V_w] \qquad w = L, H$$

and since $V_L > V_H$,

$$A(x) = [E_I(c|x), u - V_L] \cup [c_H, u]$$

- Therefore, in equilibrium

$$\Omega_L = \{(p, x) : p \in [E_I(c|x), u - V_L]\}$$
$$\Omega_H = \Omega_L \cup [c_H, u - V_H].$$

hence

$$\Omega_L = \Omega_H \qquad \text{if } V_H \geq u - c_H.$$

- Recall $E_I(c|x)$ denotes interim cost

$$E_I(c|x) = \beta_I(x)c_H + [1 - \beta_I(x)]c_L$$

- Define $x_* = x_*(B, A)$

$$\begin{cases} E_I(c|x_*) = u - V_L & \text{if} \quad V_L \geq u - E_I(c|\overline{x}) \\ \overline{x} & \text{if} \quad V_L < u - E_I(c|\overline{x}) \end{cases}$$

- Equilibrium is of the form
  - L searches till encounters (x, p) s.t.
    $x \leq x_*$
    $p \in [E_I(c|x), E_I(c|x_*)]$

  - H stops after same (x, p) and also after (x, p) s.t. $p \in [c_H, u - V_H]$

Information Aggregation

- To what extent do prices aggregate information when s is small?
  - Maximal aggregation if price paid by buyer w is close to c_w;
  - Minimal when both buyer types pay the same price(s).
- Auction literature investigated info aggregation in CV auction (Wilson(1977) and Milgrom(1979)). Milgrom's result in the 1st price auction version of our model:
  $$p_w \to c_w \text{ when } \#(\text{bidders}) \to \infty \text{ iff } \lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} = \infty.$$
- Here counterpart of increasing number of bidders is small s.
- Sequence $s^k \to 0$ & associated equilibrium sequence (B^k, A^k)
- Ω_w^k, x_*^k, V_w^k, n_w^k, $E_I^k()$, etc. magnitudes associated with (B^k, A^k)
- p_w^k = expected price paid by w in (B^k, A^k); $\overline{p}_w = \lim_{k \to \infty} p_w^k$
- S_w^k = w's expected search cost w in (B^k, A^k); $\overline{S}_w = \lim_{k \to \infty} S_w^k$.

Why expect information aggregation?

Intuitively, the good type L might search till it generates a low enough signal that will enable trading at relatively low price. If it is too costly for H to mimic this behavior, and it settles quickly for c_H the prices might aggregate information. If however H mimics L's behavior prices may fail to aggregate information.

Observations on equilibrium

- Suppose $V_L^k \geq u - E_I^k [c|\bar{x}]$. From $u - E_I^k [c|x_*^k] = V_L^k$, we get

$$\frac{s^k}{\Pi_L \left(\Omega_L^k\right)} = E_I^k (c|x_*) - p_L^k. \tag{1}$$

- Spelling it out,

$$s^k = \int_{\underline{x}}^{x_*^k} \left(\int_{E_I^k[c|x]}^{u-V_L^k} \left(u - V_L^k - p \right) g\left(p \right) dp \right) f_L\left(x \right) dx. \tag{2}$$

Claim: **(i)** $\lim x_*^k = \underline{x}$; **(ii)** $\lim p_L^k = \lim E_I^k(c|\underline{x})$; **(iii)** $\overline{S}_L = 0$

- That is: L searches till generates the best signal; gets the lowest price associated with that signal; incurs negligible search cost.

Proof: Suppose $\lim x_*^k > \underline{x}$.

$\implies \lim \frac{n_L}{n_H} (\equiv \lim \frac{\Pi_H(\Omega_H^k)}{\Pi_L(\Omega_L^k)})$ is bounded since it is just boundedly more costly for H to mimic L.

$\implies \lim E_I^k [c|x_*^k] > \lim E_I^k [c|\underline{x}] \implies \lim \text{RHS(2)} > 0$ – contradiction.

$\implies \lim x_*^k = \underline{x}$.

Since $p_L^k \in [E_I^k(c|\underline{x}), E_I^k(c|x_*^k)]$, $\bar{p}_L \equiv \lim p_L^k = \lim E_I^k [c|\underline{x}]$.

Hence, by (1), $\lim \frac{s^k}{\Pi_L(\Omega_L^k)} = 0$. ∎

- Not full proof: remaining case $V_L^k < u - E_I^k [c|\bar{x}]$

Boundedly informative signals

Proposition: Suppose $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} < \infty$. Consider $s^k \to 0$ and an associated sequence of equilibria $\left\{ B^k, A^k \right\}$. Then,

$$\bar{p}_L = \rho_L c_L + \rho_H c_H = \bar{p}_H.$$

- No information aggregation: price = EX-ANTE expected cost.
- As we know L searches for a signal below x_*^k. Here, H mimics L and both end up trading after the same low signals and in the limit at the same low price.
- Different from outcome of CV auction with many bidders (later).

Proof: From observations above

$$\lim_{k \to \infty} \left(u - V_L^k - E_I^k \left[c | \underline{x} \right] \right) = 0.$$

Since $V_H^k < V_L^k$, $\lim V_H^k \leq \lim V_L^k$. Also,

$$
\begin{aligned}
V_H^k &\geq E_{(p,x)} \left[u - p \mid (p, x) \in \Omega_L^k, H \right] - \frac{s^k}{\Pi_H \left(\Omega_L^k \right)} \\
&\geq V_L^k - \frac{\Pi_L \left(\Omega_L^k \right)}{\Pi_H \left(\Omega_L^k \right)} \frac{s^k}{\Pi_L \left(\Omega_L^k \right)} \geq V_L^k - \frac{F_L \left(x_*^k \right)}{F_H \left(x_*^k \right)} \frac{s^k}{\Pi_L \left(\Omega_L^k \right)} \\
&\geq V_L^k - \frac{f_L \left(\underline{x} \right)}{f_H \left(\underline{x} \right)} \frac{s^k}{\Pi_L \left(\Omega_L^k \right)} \to \lim V_L^k
\end{aligned}
$$

where 2nd inequality owes to $u - p \geq V_L^k$ for all $(p, x) \in \Omega_L^k$, and final step owes to previous observation $\lim \frac{s^k}{\Pi_L \left(\Omega_L^k \right)} = 0$. Therefore, $\lim V_H^k \geq \lim V_L^k$ and the previous observation $\lim V_L^k = u - \lim E_I^k \left[c | \underline{x} \right]$ implies $\lim V_H^k = u - \lim E_I^k \left(c | \underline{x} \right)$ as well.

Since in the limit BOTH H and L trade after $\underline{x}$, it must be that $\lim E_I^k \left(c | \underline{x} \right)$ is equal to the ex-ante cost $\rho_L c_L + \rho_H c_H$. $\blacksquare$

- The argument in the end of the proof, concerning why $\lim E_I^k(c|\underline{x})$ is equal to the ex-ante cost $\rho_L c_L + \rho_H c_H$, can be explained using different words as follows.

- Recall $\beta_I(x)$ can be written as

$$\beta_I(x) = \frac{\rho_H}{\rho_H + \rho_L \frac{f_L(x)n_L}{f_H(x)n_H}}$$

where $\frac{f_L(x)n_L}{f_H(x)n_H}$ is the combined likelihhod ratio of being sample and the signal.

- Since $\frac{n_L^k}{n_H^k} = \frac{\Pi_H(\Omega_H^k)}{\Pi_L(\Omega_L^k)} = \frac{F_H(x_*^k)}{F_L(x_*^k)} \rightarrow \frac{f_H(\underline{x})}{f_L(\underline{x})}$, the "sampling effect" $\frac{n_H^k}{n_L^k}$ exactly offests the signal effect $\frac{f_L(\underline{x})}{f_H(\underline{x})}$ in the formula of $\beta_I(\underline{x})$ so $\beta_I(\underline{x}) = \rho_H$ and hence $E_I[c|\underline{x}] = \rho_L c_L + \rho_H c_H$

Information aggregation

- Assume that $\lim_{x \to \underline{x}} \frac{\frac{d}{dx}\left(\frac{f_L(x)}{f_H(x)}\right)}{\frac{f_L(x)}{F_L(x)}}$ exists and let

$$\lambda \triangleq - \lim_{x \to \underline{x}} \frac{\frac{d}{dx}\left(\frac{f_L(x)}{f_H(x)}\right)}{\frac{f_L(x)}{F_L(x)}}$$

  which may be ∞ as well (the case in which this limit does not exist is taken up later).

- λ is a measure of the informativeness of the signal related to the rate of change of the likelihood ratio near the lower bound $\underline{x}$.

PROPOSITION: Suppose the limit λ exists. Consider a sequence $s^k \to 0$ and a sequence $\left(B^k, A^k\right)$ of corresponding equilibria.

$$\bar{p}_L = \begin{cases} (1 - \frac{1}{\lambda})c_L + \frac{1}{\lambda}c_H & \text{if} \quad \lambda \in [\frac{1}{\rho_H}, \infty], \\ \rho_L c_L + \rho_H c_H & \text{if} \quad \lambda \leq \frac{1}{\rho_H}, \end{cases}$$

$$\bar{p}_H = \begin{cases} \frac{1}{\lambda}\frac{\rho_L}{\rho_H}c_L + (1 - \frac{1}{\lambda}\frac{\rho_L}{\rho_H})c_H & \text{if} \quad \lambda \in [\frac{1}{\rho_H}, \infty], \\ \rho_L c_L + \rho_H c_H & \text{if} \quad \lambda \leq \frac{1}{\rho_H}. \end{cases}$$

The conclusion of the proposition rephrased:

$$\lambda \leq \frac{1}{\rho_H}: \quad \text{no aggregation} \quad \bar{p}_L = \bar{p}_H = \rho_L c_L + \rho_H c_H$$

$$\lambda \geq \frac{1}{\rho_H}: \quad \text{partial aggregation} \quad \bar{p}_L = (1 - \frac{1}{\lambda})c_L + \frac{1}{\lambda}c_H \neq \bar{p}_H$$

- Boundedly informative signal, $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} < \infty$,
  $\to \lambda = 0 \to$ no aggregation.
- $\lambda \neq 0$ means unboundedly informative signal, $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} = \infty$,
  but not necessarily perfect aggregation
    - Small $\lambda \to$ no aggregation, $\bar{p}_L = \bar{p}_H$
    - Larger $\lambda \to$ partial aggregation, $c_L < \bar{p}_L < \bar{p}_H < c_H$.
    - Very large $\lambda \to$ nearly perfect aggregation, $\bar{p}_w \approx c_w$.
- Reminder about corresponding auction:
    - full aggregation whenever $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} = \infty$ w/o further distinctions.
    - partial aggregation when $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} < \infty$
- No aggregation price$= \rho_H c_H + \rho_L c_L =$ ex-ante expected cost.
- Extent of info aggregation increases in signal informativeness as
  measured by λ.

Equilibrium description near the limit

- No aggregation region: Both L & H search for $x \leq x_*$

- Partial aggregation region:
  - L searches for $x \leq x_*$.
  - H searches for $x \leq x_*$ and settles for $p \in [c_H, V_H]$
    (in limit H indifferent between search for $x \leq x_*$ & settle for c_H.
  - H's "mix" of search and settlement is determined endogenously by the relative "sizes" of the $[\underline{x}, x_*]$ and $[c_H, V_H]$ regions.
  - H's "mix" determines $E_I(c|x)$ which in turn determine x_* (via L's behavior) which in turn determines H's "mix".

- When λ is very large, the partial aggregation is nearly perfect aggregation in that H most likely settles for price near c_H, the prices accepted after $x \leq x_*$ are near c_L.

Alternative formula for lambda

- Assume $\lim_{x_* \to \underline{x}} \int_{\underline{x}}^{x_*} \left(\frac{f_L(x)}{f_H(x)} - \frac{f_L(x_*)}{f_H(x_*)} \right) \frac{f_L(x)}{F_L(x_*)} dx$ exists and let

$$\lambda \triangleq \lim_{x_* \to \underline{x}} \int_{\underline{x}}^{x_*} \left(\frac{f_L(x)}{f_H(x)} - \frac{f_L(x_*)}{f_H(x_*)} \right) \frac{f_L(x)}{F_L(x_*)} dx$$

  which may be ∞ as well.

- If $\lim_{x \to \underline{x}} \frac{\frac{d}{dx}\left(\frac{f_L(x)}{f_H(x)} \right)}{\frac{f_L(x)}{F_L(x)}}$ exists the new λ coincides with the previous one.

- But it may exist under broader circumstances and still the proposition holds.

- This alternative definition also facilitates consideration of limit non-existence.

- If limit λ does not exist let,

$$\overline{\lambda} = \lim_{x_* \to \underline{x}} \sup \int_{\underline{x}}^{x_*} \left(\frac{f_L(x)}{f_H(x)} - \frac{f_L(x_*)}{f_H(x_*)} \right) \frac{f_L(x)}{F_L(x_*)} dx \text{ and } \underline{\lambda} = \lim \inf$$

PROPOSITION:(i) Consider a sequence $s^k \to 0$ and a sequence (B^k, A^k) of corresponding equilibria s.t. $\bar{p}_L = \lim p_L^k$ exists. Then $\exists \lambda \in [\underline{\lambda}, \overline{\lambda}]$ such that.

$$\bar{p}_L = \begin{cases} (1 - \frac{1}{\lambda})c_L + \frac{1}{\lambda}c_H & \text{if} \quad \lambda \in (\frac{1}{\rho_H}, \infty], \\ \rho_L c_L + \rho_H c_H & \text{if} \quad \lambda \le \frac{1}{\rho_H}, \end{cases}$$

$$\bar{p}_H = \begin{cases} \frac{1}{\lambda}\frac{\rho_L}{\rho_H}c_L + (1 - \frac{1}{\lambda}\frac{\rho_L}{\rho_H})c_H & \text{if} \quad \lambda \in (\frac{1}{\rho_H}, \infty], \\ \rho_L c_L + \rho_H c_H & \text{if} \quad \lambda \le \frac{1}{\rho_H}. \end{cases}$$

(ii) For any $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, $\exists$a sequence $s^k \to 0$ and a sequence (B^k, A^k) of corresponding equilibria s.t. $\bar{p}_L = \lim p_L^k$ and $\bar{p}_H = \lim p_H^k$ exist and are as above

Examples (SKIP!)

- Let $\lambda < \infty$.

$$F_L(x) = \begin{cases} 1 & \text{for} \quad x \geq r \\ e^{-\frac{1}{\lambda}\left(\frac{1}{x} - \frac{1}{r}\right)} & \text{for} \quad x \in (0, r] \\ 0 & \text{for} \quad x = 0 \end{cases}$$

$$F_H(x) = \begin{cases} 1 & \text{for} \quad x \geq r \\ \frac{g_L}{g_H} \int_0^x \frac{t}{1-t} f_L(t) dt & \text{for} \quad x \in (0, r] \\ 0 & \text{for} \quad x = 0 \end{cases}$$

  where $r > g_H$ solves $\frac{g_L}{g_H} \int_0^r \frac{x}{1-x} f_L(x) dx = 1$, i.e., implied F_H is a CDF.

- λ parameter above is λ of propsition.
- Since $\lambda < \infty$., no full aggregation but arbitrarily close to it when λ very large.
- F_L with lower λ stoch. dominates F_L with higher λ
- Here $\frac{g_H f_H(x)}{g_H f_H(x) + g_L f_L(x)} = x$. So stoch. dominated F_L is more infomative.

Examples–Continued

- Suppose signals are from $x \in (-\infty, -r)$

$$F_L(x) = \mu e^{x+r}$$
$$F_H(x) = \int_{-\infty}^{-x} (-t)^{-\alpha} e^{t+r} dt$$

for $x \leq -r$, for some $r \geq 0$ solving $\int_{-\infty}^{-r} (-x)^{-\alpha} e^{x+r} dx = 1$

- Here the λ is

$$\lambda = \begin{cases} 0 & \text{if} \quad \alpha < 1 \\ \frac{1}{\mu} & \text{if} \quad \alpha = 1 \\ \infty & \text{if} \quad \alpha > 1 \end{cases}$$

Welfare

- Expected surplus fully determined by expected search cost.
- Recall $\overline{S}_w$, $w = L, H$, denotes lim(expected search costs).

Proposition: (i) $\overline{S}_L = 0$ in all cases;
(ii)
$$\overline{S}_H = \begin{cases} \frac{1}{\lambda} \frac{\rho_L}{\rho_H^2}(1 - \frac{\rho_L}{\rho_H}\frac{1}{\lambda})(c_H - c_L) & \text{if} \quad \lambda \in [\frac{\rho_L}{\rho_H^2}, \infty] \\ (c_H - c_L)\lambda\rho_H^2 & \text{if} \quad \lambda < \frac{\rho_L}{\rho_H^2} \end{cases}$$

- Thus, If $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} < \infty$, then $\overline{S}_H = 0$;
- Limit **efficient** if $\overline{S}_w = 0$, $w = L, H$. Limit ε-**efficient** if $\overline{S}_w < \varepsilon$.
- For any $\varepsilon > 0$, limit is ε-efficient when λ is sufficiently small (uninformative) or sufficiently large (informative). [When $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} < \infty$ limit is fully efficient .]
- Nonmonotonicity in signal informativeness: If $\lambda < \frac{\rho_L}{\rho_H^2}$, more informative signal technology decreases surplus.

Welfare–Continued

- Can use the above to construct examples in which welfare decreases in search cost.

- Can modify to introduce efficiency considerations in trade volume. Simplest such modification $c_H > u$.

Comparison to Auctions

- Auction: n bidders (independent of state) first (lowest) price.
- Look at limit as $n \rightarrow \infty$
  - Milgrom's result: winning bid $\rightarrow$ true cost, iff $\lim_{x \rightarrow \underline{x}} \frac{f_L(x)}{f_H(x)} = \infty$
  - If $\lim_{x \rightarrow \underline{x}} \frac{f_L(x)}{f_H(x)} < \infty$, partial aggregation: $p_L \neq p_H$ but $p_w \neq c_w$.
- In search model:
  - number of "bidders" endogenous: $s \rightarrow 0$ counterpart of $n \rightarrow \infty$.
  - when $\lim_{x \rightarrow \underline{x}} \frac{f_L(x)}{f_H(x)} = \infty$ equilibrium price not always near true cost when $s \rightarrow 0$, but only when λ large enough (i.e., additional informativeness requirement is met).
  - When $\lim_{x \rightarrow \underline{x}} \frac{f_L(x)}{f_H(x)} < \infty$, no aggregation at all.
- Conclusion: info aggregation more difficult in search than in auction.

Winner's curse perspective

- In monotone equilibrium of auction

$$\Pr(H \mid \text{winning}, x) = \frac{\rho_H}{\rho_H + \rho_L \frac{f_L(x)}{f_H(x)} \frac{[1-F_L(x)]^{n-1}}{[1-F_H(x)]^{n-1}}}$$

- $\frac{f_L(x)}{f_H(x)} =$"signal effect"; $\frac{[1-F_H(x)]^{n-1}}{[1-F_L(x)]^{n-1}} =$ "winner's curse effect."
- $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} = \infty \Rightarrow$ for low x, $\frac{f_L(x)}{f_H(x)} >> \frac{[1-F_H(x)]^{n-1}}{[1-F_L(x)]^{n-1}}$.
- How about for x that is "likely to win"?
    - x_n has "reasonable winning probability" in monotone equilibrium with n
    - I.e., $[1-F_L(x_n)]^n \geq \varepsilon > 0 \Rightarrow \lim \frac{[1-F_L(x_n)]^n}{[1-F_H(x_n)]^n} \geq \varepsilon \Rightarrow$
      $\lim \frac{f_L(x_n)}{f_H(x_n)} \frac{[1-F_L(x_n)]^n}{[1-F_H(x_n)]^n} = \infty$

      $\Rightarrow \Pr(H \mid \text{winning auction}, x_n) \approx 0.$

Winner's curse perspective-continued

- In search version just being sampled already implies a sort of winner's curse. Here

$$\Pr(H \mid \text{being sampled}, x) = \beta_I(x) = \frac{\rho_H}{\rho_H + \rho_L \frac{f_L(x)}{f_H(x)} \frac{n_L}{n_H}}$$

$\frac{f_L(x)}{f_H(x)} =$ "signal effect"; $\frac{n_L}{n_H} =$ "winner's curse effect."

- While in auction $\frac{f_L(x)}{f_H(x)}$ prevails, here winner's curse effect $\frac{n_L}{n_H}$ might offset $\frac{f_L(x)}{f_H(x)}$ even when $\lim_{x \to \underline{x}} \frac{f_L(x)}{f_H(x)} = \infty$.

- E.g., if both H and L search till $x \leq x_*$, which may arise in equilibrium, then $\lim_{x_* \to \underline{x}} \frac{f_L(x_*)}{f_H(x_*)} \frac{n_L}{n_H} = \lim_{x_* \to \underline{x}} \frac{f_L(x_*)}{f_H(x_*)} \frac{F_H(x_*)}{F_L(x_*)} = 1$ and $\beta_I(x) \to \rho_H$.

Discussion-modeling

- The random proposals model avoids both Diamond's Paradox and multiplicity due to freedom of off path beliefs that would arise if the privately informed buyer gets to offer.
- The draw back is the artficial nature of the random proposal model. But is it really more artficial than other bargaining models?

Alternative models yielding the same results

- **Buyer makes a TOL offer in each encounter.** Multiplicity is refined by invoking the undefeated equilibrium refinement (Mailath-Okuno-Postlewaite, 1993) in the bargaining game after any x.

- **Bertrand.** Each period the buyer samples TWO sellers who observe the same signal and simultaneously offer prices. The buyer then either trades with the seller who offered the lower price or continues to search. All else remains the same.

Extensions

- **Many types.** Believe that can be extended to many (but finite) number of types.

- **Buyer does not know own type:** Might require work but might not be very different.

- **Pesendorfer-Swinkles:** What would be the analog of their model in the search environment? And how it would behave with respect to information aggregation? This question occurred to me just recently. I do not know how to model it and what might happen.

- **Application.** Loans.